PSO 8

Graph

I can’t wait for spring break

Any fun plans

Unfortunately busy week so no slides today :(

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JUST KIDDING

Question 1

(Adjacency-list Representation)

1. Given an adjacency-list representation of a directed graph, how long does it take to compute the out-degree of a vertex? How long does it take to compute the in-degree of a vertex?

2. The transpose of a directed graph $G = (V, E)$ is the graph $G^\top = (V, E^\top)$, where $E^\top := \{(v, u) : (u, v) \in E\}$. In other words, $G^\top$ is G with all its edges reversed. Describe an efficient algorithm for computing $G^\top$ from G for the adjacency-list representations of G and analyze the runtime of your algorithm.

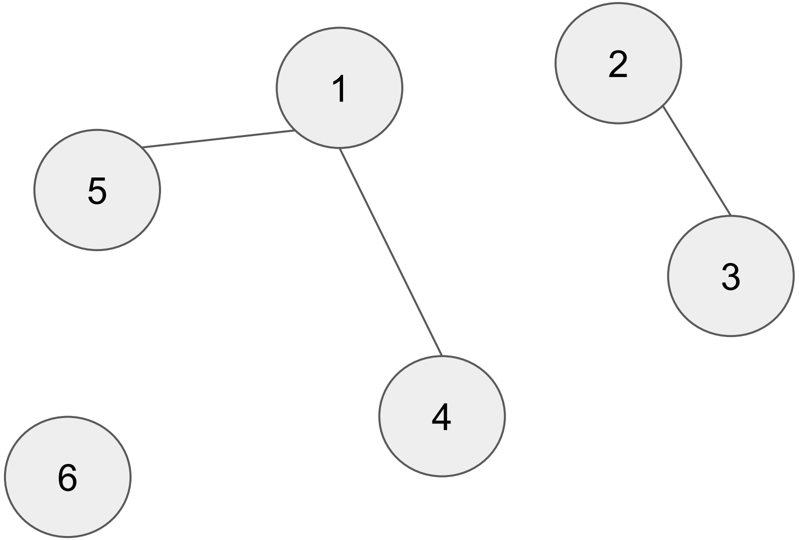
3. The square of a directed graph $G = (V, E)$ is the graph $G^2 = (V, E^2)$, where $(u, v) \in E^2$ if and only if G contains a path with at most two edges between u and v. Describe an efficient algorithm for computing G^2 from G for the adjacency-list representations of G and analyze the runtime of your algorithm.

What is an adjacency list?

Adjacency list

A linked list per vertex

E.g. if undirected..

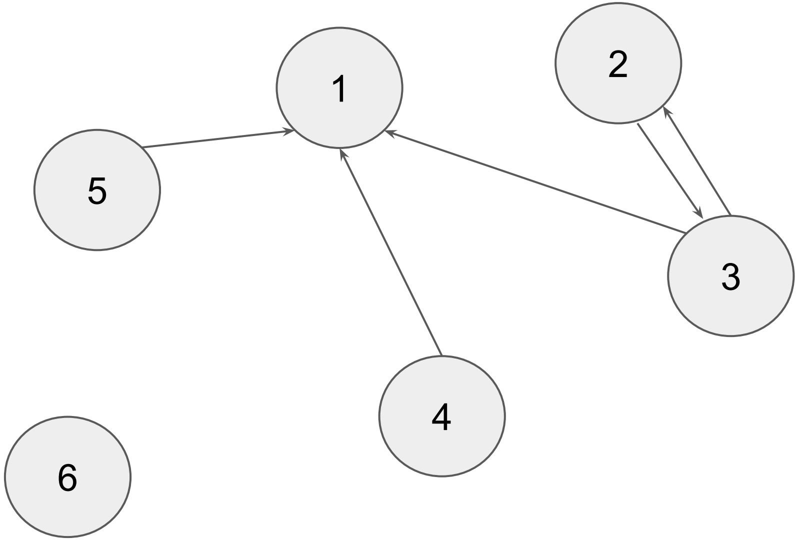


Vertex	Adjacency
1	
2	
3	
4	
5	
6	

Adjacency list

A linked list per vertex

E.g. if **directed**..



Vertex	Adjacency (*points to*)
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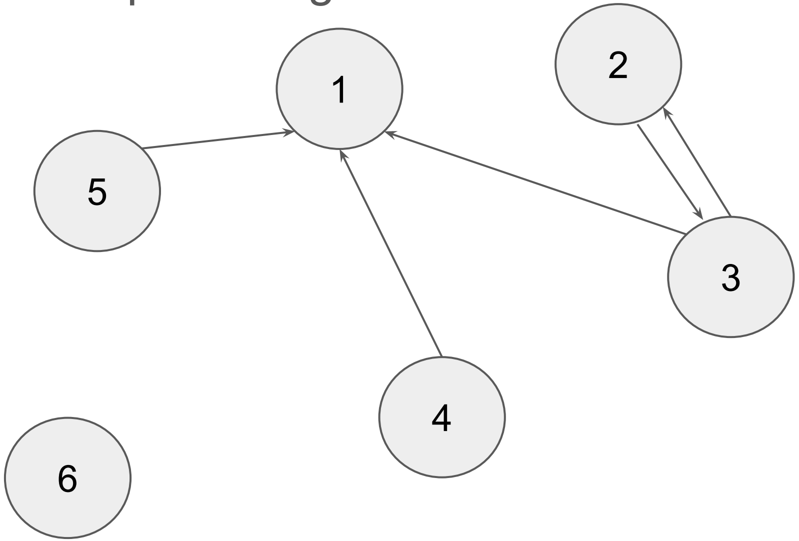
1. Given an adjacency-list representation of a directed graph, how long does it take to compute the out-degree of a vertex? How long does it take to compute the in-degree of a vertex?

Example: indeg. of 1?



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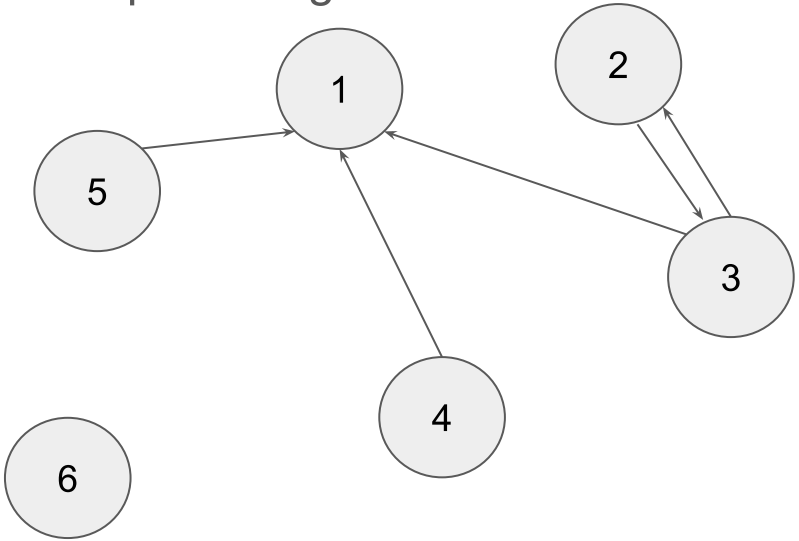

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O(|Adjaceny list|)

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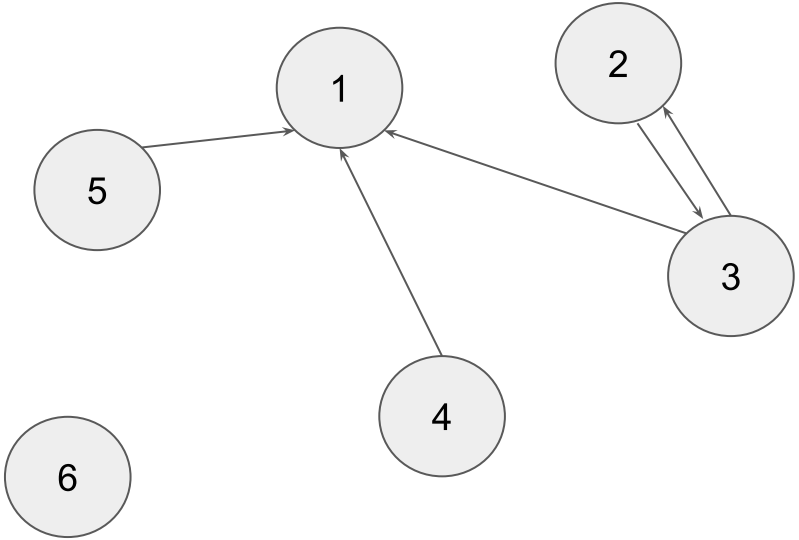
Try counting the indegree for v = 1



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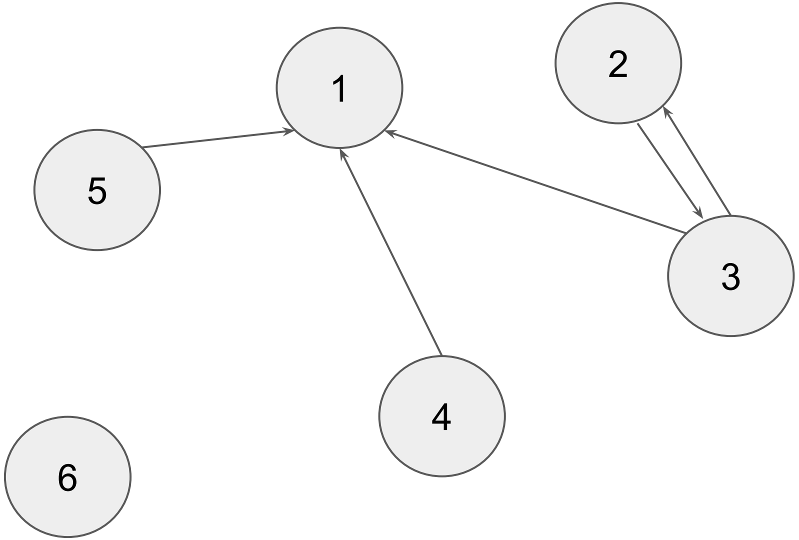
1. Given an adjacency-list representation of a directed graph, how long does it take to compute the out-degree of a vertex? How long does it take to compute the in-degree of a vertex?

For indeg. of vertex i:

Iterate over each vertex list other than i,
Count for every instant of i you see

O(|E|) time



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Let's see how this looks..

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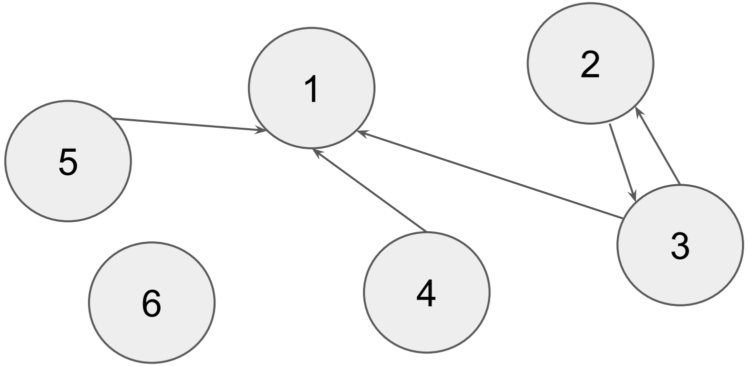

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We want to go from this

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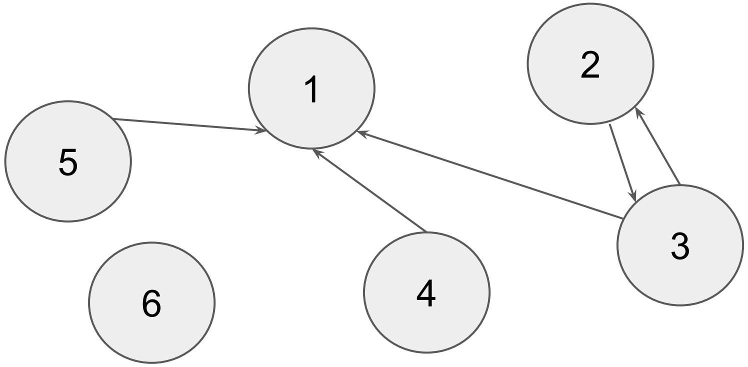

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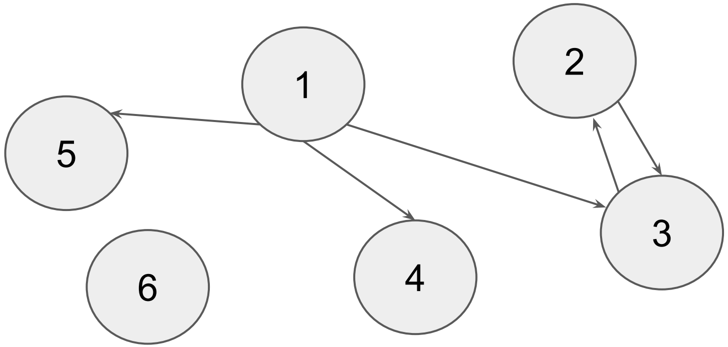
We want to go from this to this

Vertex	Adjacency (*points to*)
1	3,4
2	3
3	2
4	
5	1
6	



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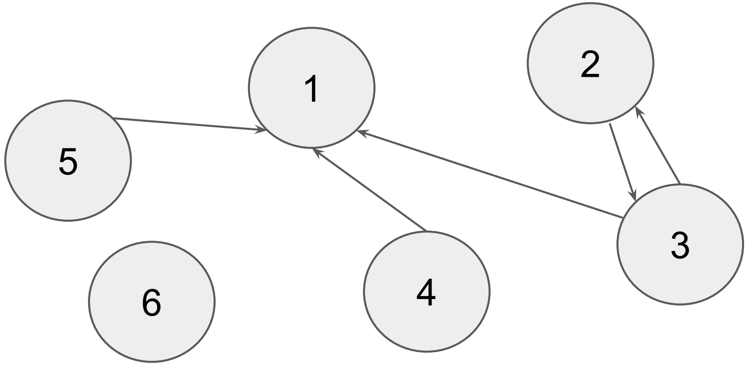

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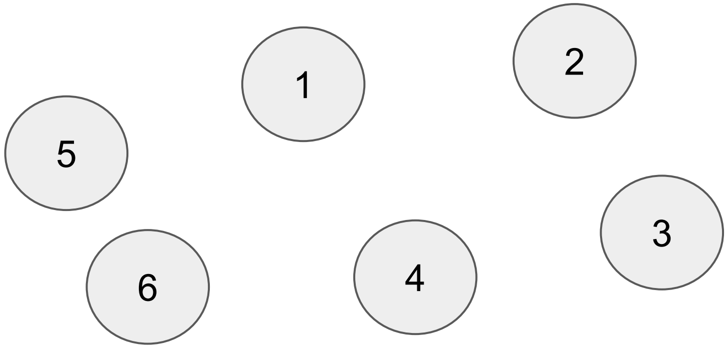
Easiest algorithm

1. Iterate through each vertex list i
2. Add the “reverse” to the new adjacency list



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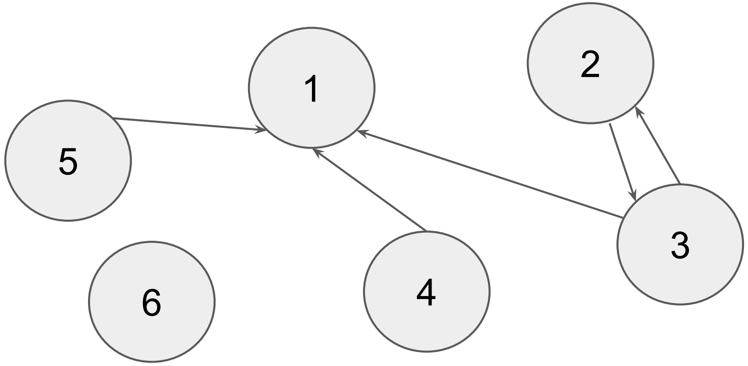

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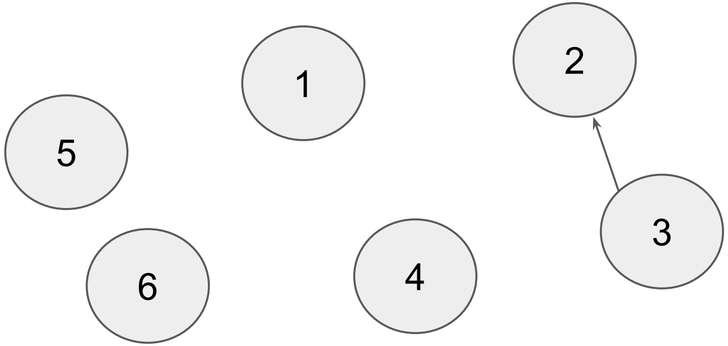

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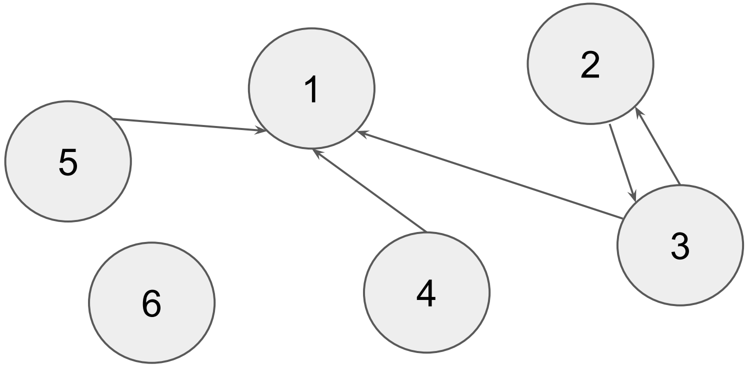

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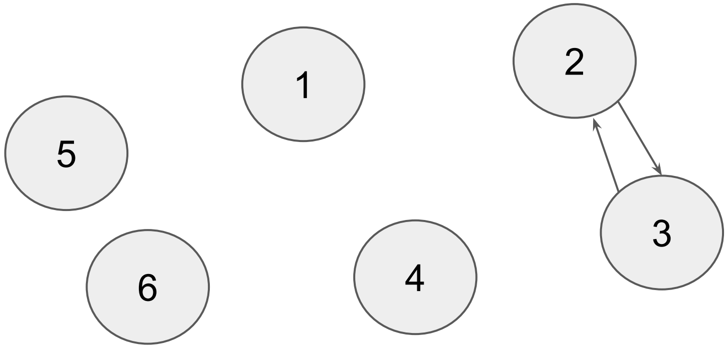

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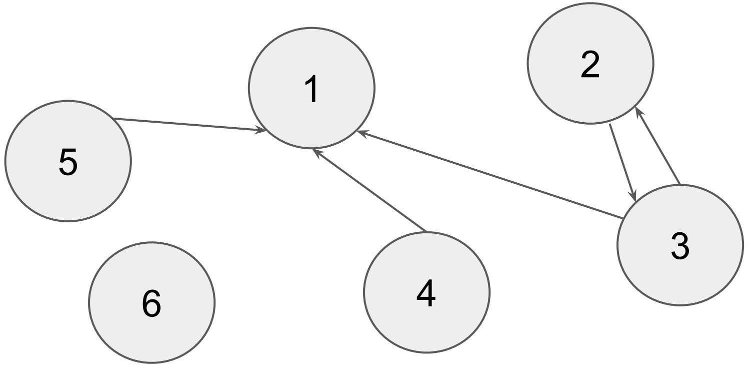

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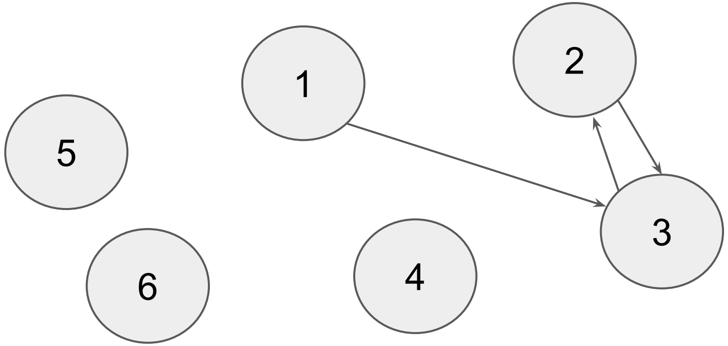

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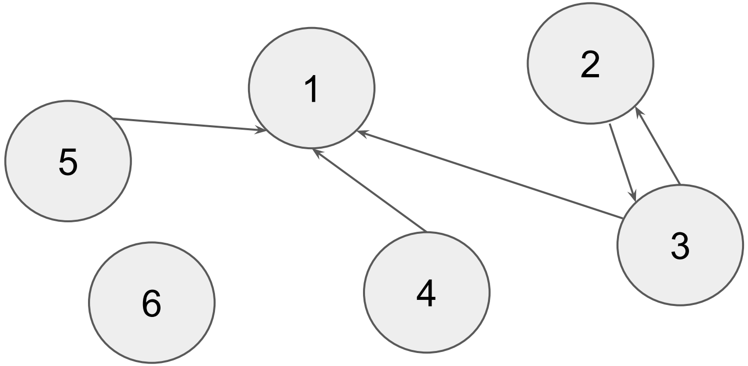

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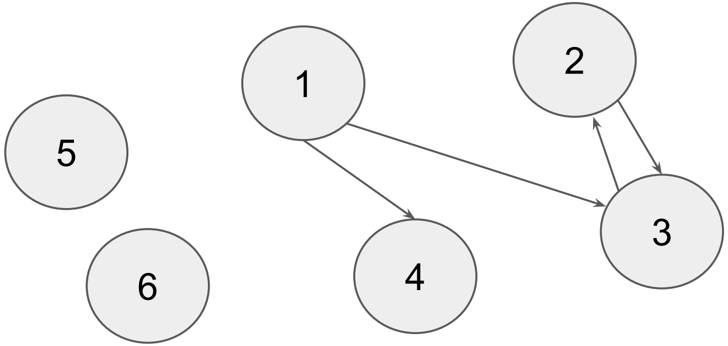

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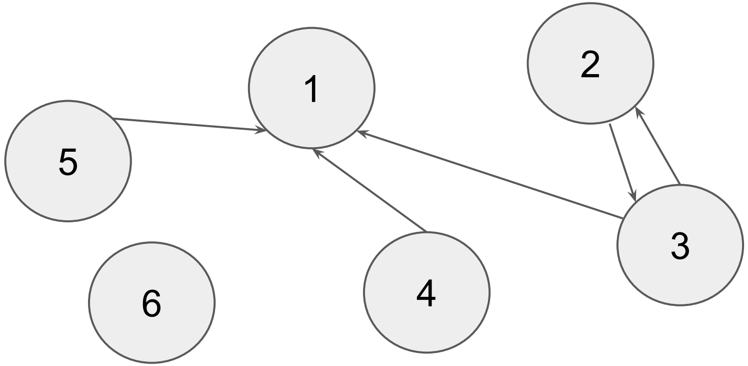

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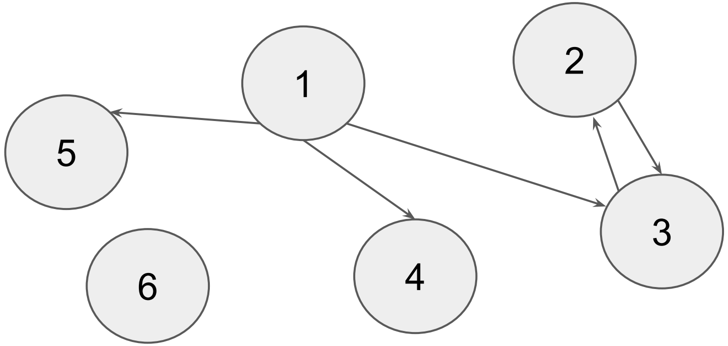
Runtime?



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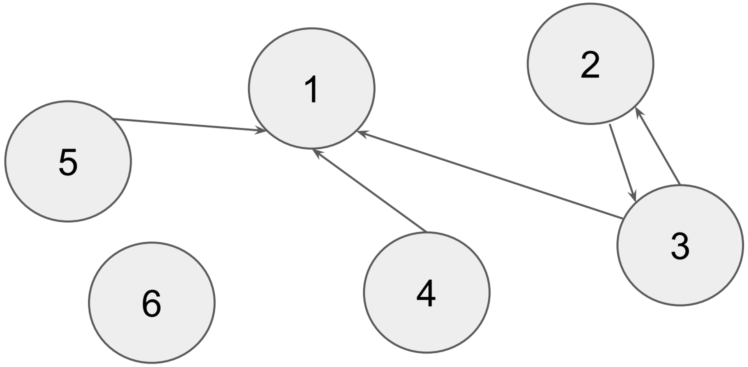

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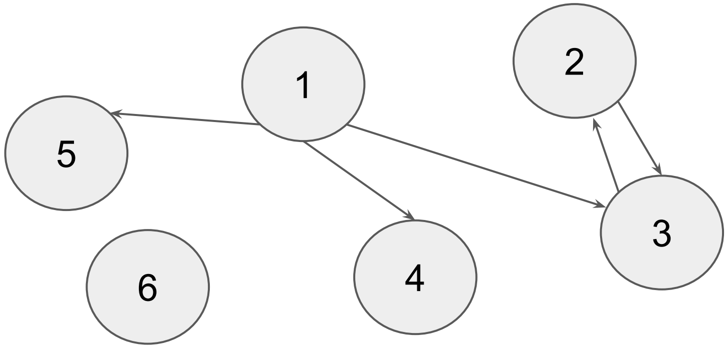
Runtime? O(|V| + |E|)



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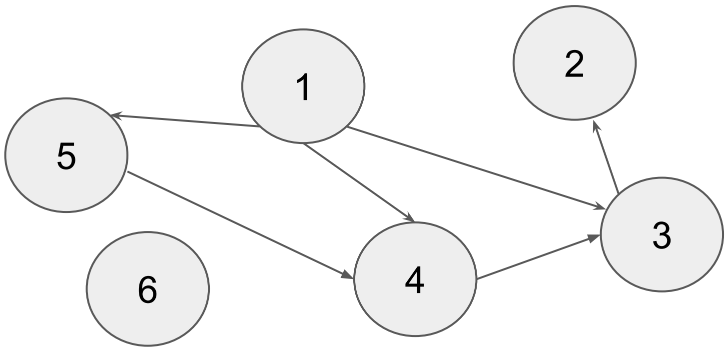
Square of this graph?



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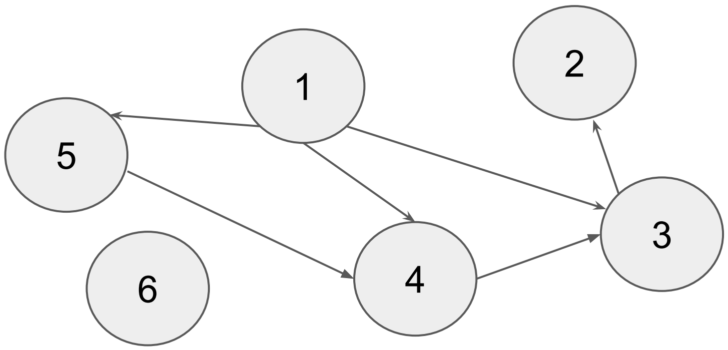

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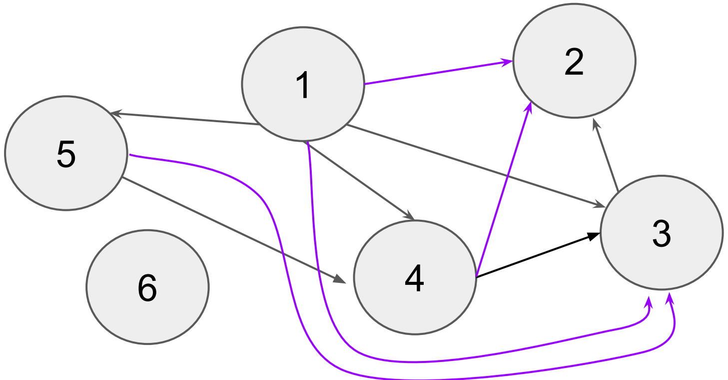

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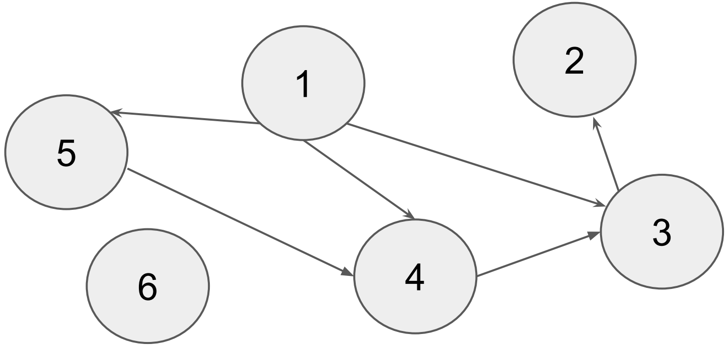
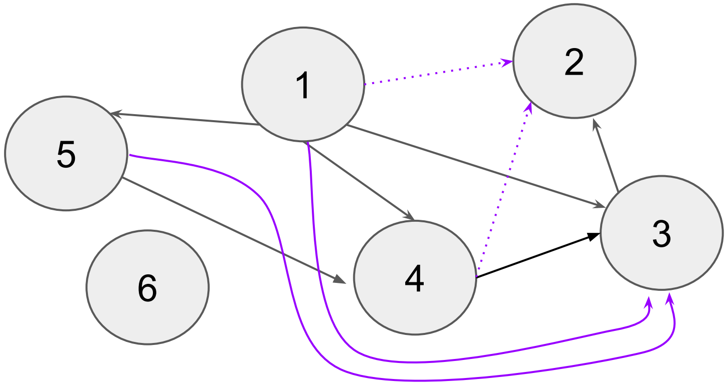
How do we get 3's edges (ignore every else for now)





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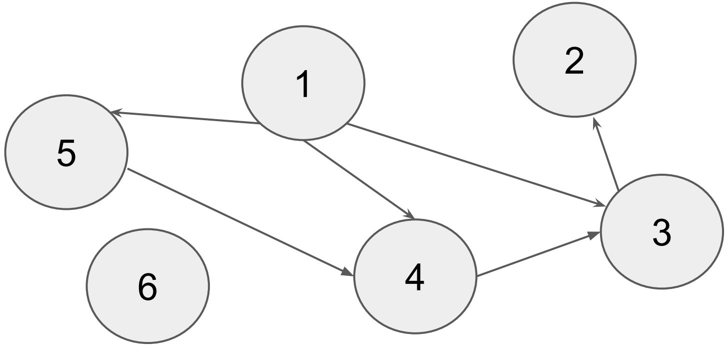

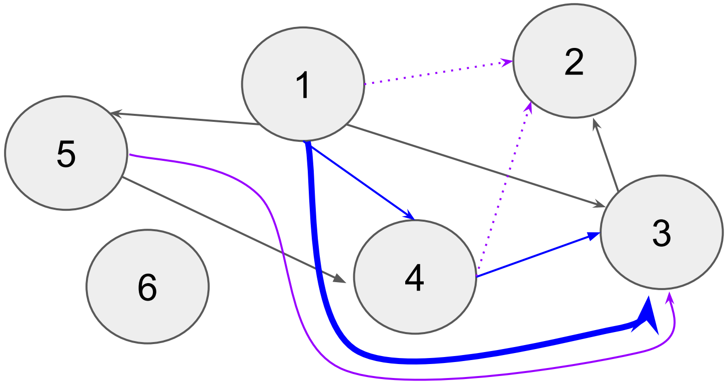

edge comes from

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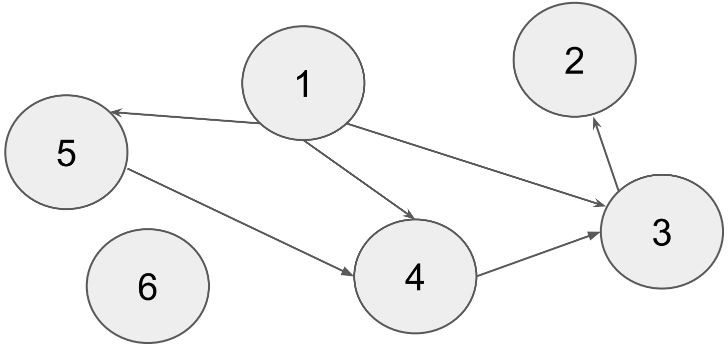

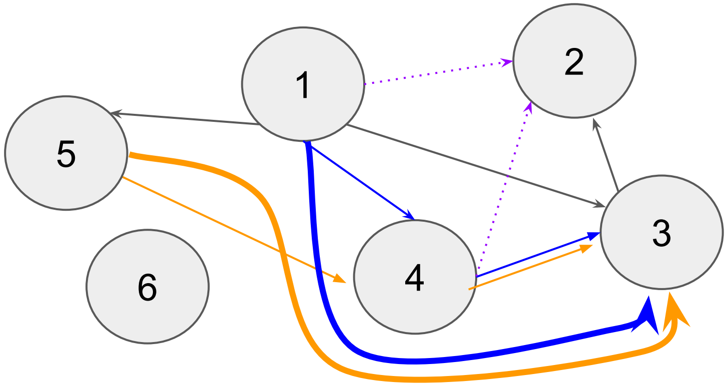

edge comes from _

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They both share a (3,4) edge

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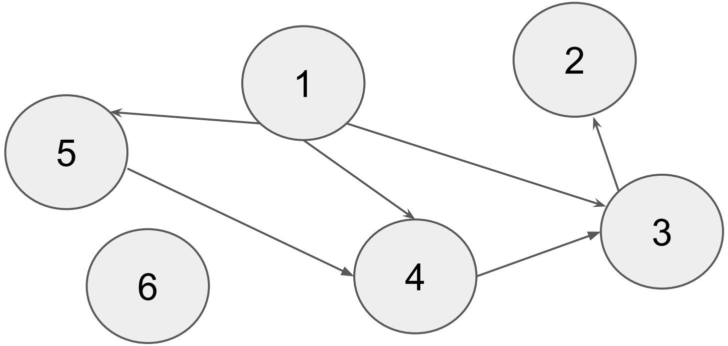

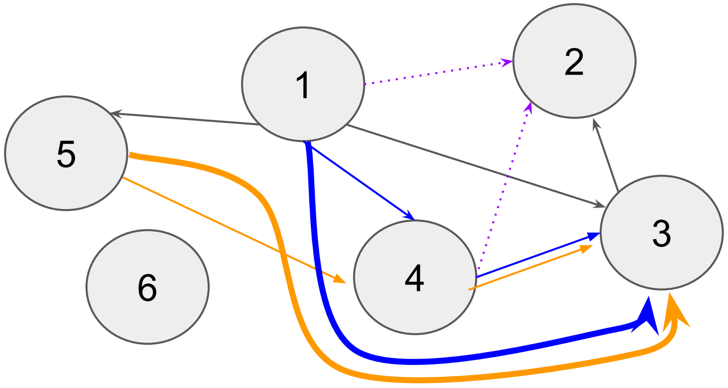

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- **Idea**: when adding edge (3,4),
  Add all edges pointing to 3

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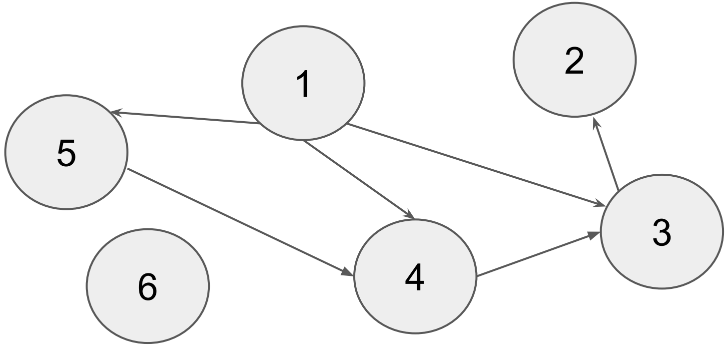

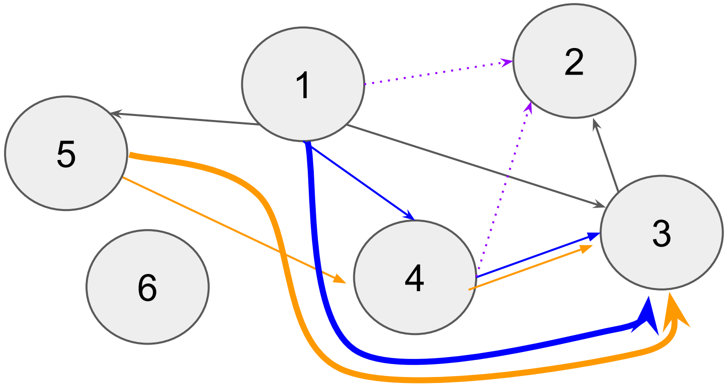

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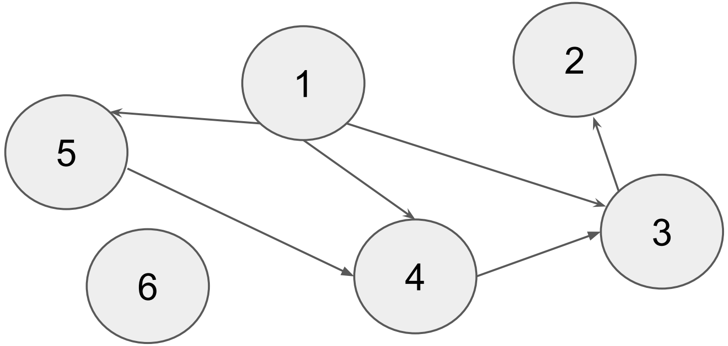
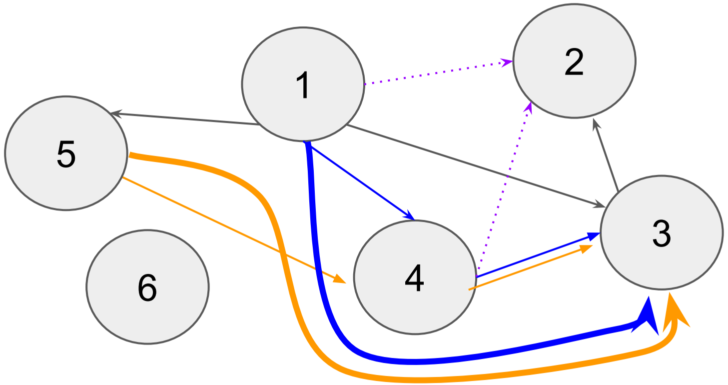
How do we get 3's edges (ignore every else for now)





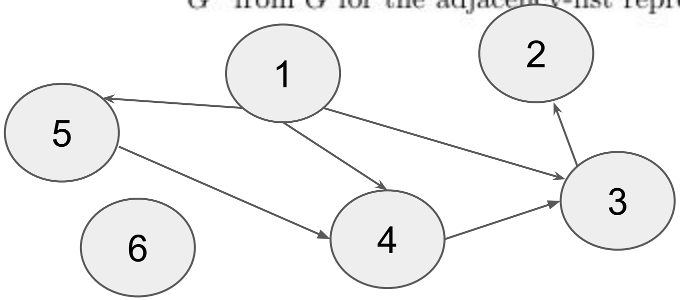
They both share a (3,4) edge

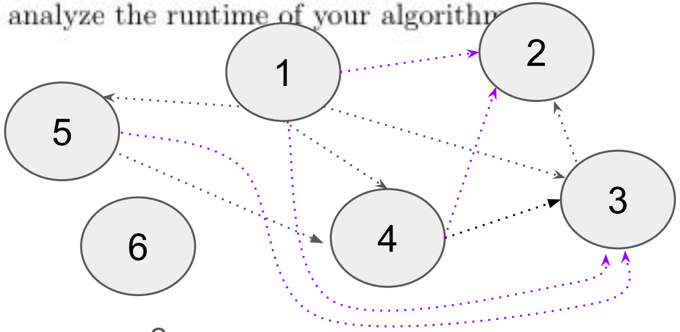
- **Idea**: when adding edge (i,j),

  Add all edges pointing to i to j (how do we get this?)

  G^T.adjList(i) is exactly this!

3. The square of a directed graph $G = (V, E)$ is the graph $G^2 = (V, E^2)$, where $(u, v) \in E^2$ if and only if G contains a path with at most two edges between u and v. Describe an efficient algorithm for computing G^2 from G for the adjacency-list representations of G and analyze the runtime of your algorithm.





G

G^T

G^2

Vertex	Adj
1	**3**,4,5
2	
3	2
4	3
5	4
6	

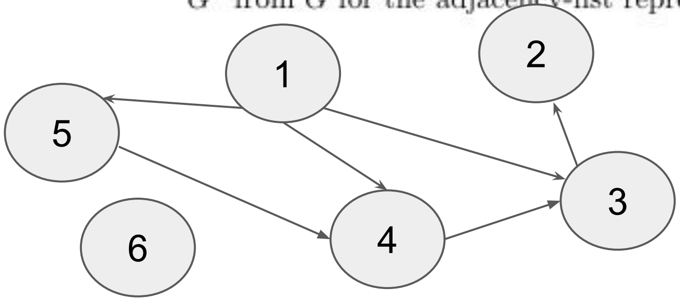
Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

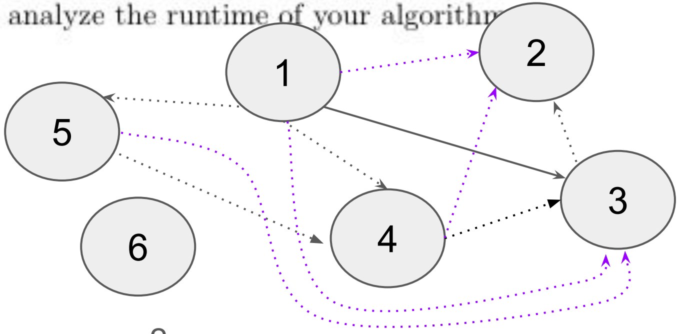
Vertex	Adj
1	
2	
3	
4	
5	
6	

Idea: when adding edge (i,j), Add all edges pointing to i to j

For i in |V|:
    For **j** in G.adjList(i):

3. The square of a directed graph $G = (V, E)$ is the graph $G^2 = (V, E^2)$, where $(u, v) \in E^2$ if and only if G contains a path with at most two edges between u and v. Describe an efficient algorithm for computing G^2 from G for the adjacency-list representations of G and analyze the runtime of your algorithm.





G^T

G^2

Vertex	Adj
1	**3**,4,5
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3
2	
3	
4	
5	
6	

Idea: when adding edge (i,j), Add all edges pointing to i to j

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For i in |V|:
    For j in G.adjList(i):
        add j to G^2.adjList(i)
```

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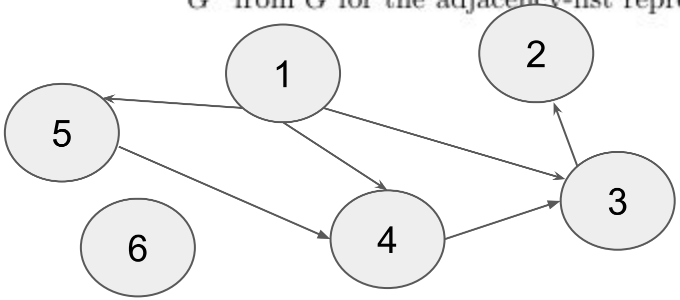
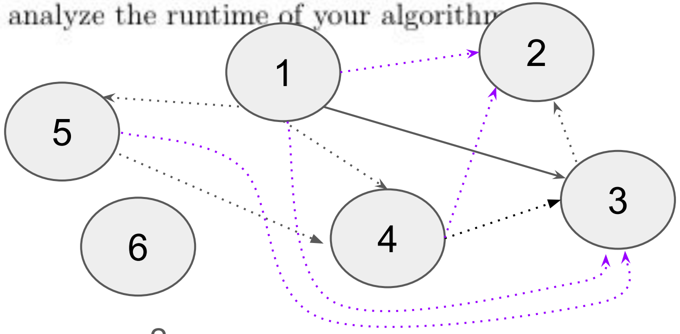


G

G^T



G^2

Vertex	Adj
1	**3**,4,5
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3
2	
3	
4	
5	
6	

Idea: when adding edge (i,j), **Add all edges pointing to i to j**

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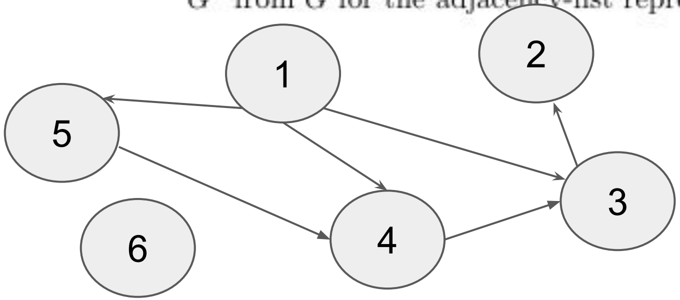

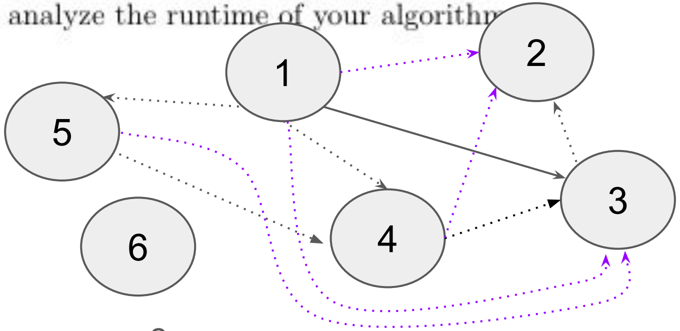

G

Vertex	Adj
1	**3**,4,5
2	
3	2
4	3
5	4
6	

G^T

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

G^2

Vertex	Adj
1	3
2	
3	
4	
5	
6	

Idea: when adding edge (i,j), **Add all edges pointing to i to j**

```
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        for k in G^T.adjList(i):
            add j to
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```

Bad example lol
Lets continue

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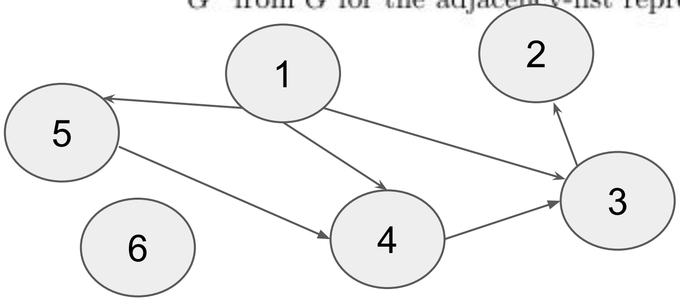
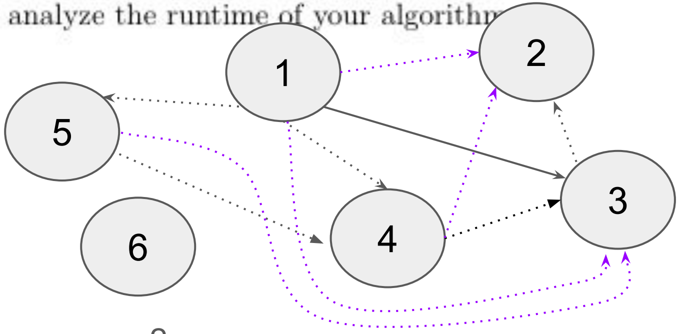


G

G^T



G^2

Vertex	Adj
1	3,**4**,5
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3
2	
3	
4	
5	
6	

Idea: when adding edge (i,j), **Add all edges pointing to i to j**

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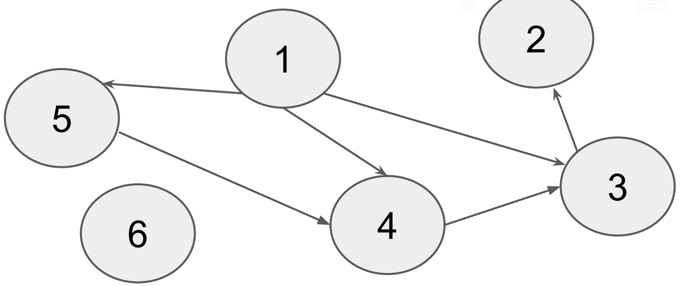
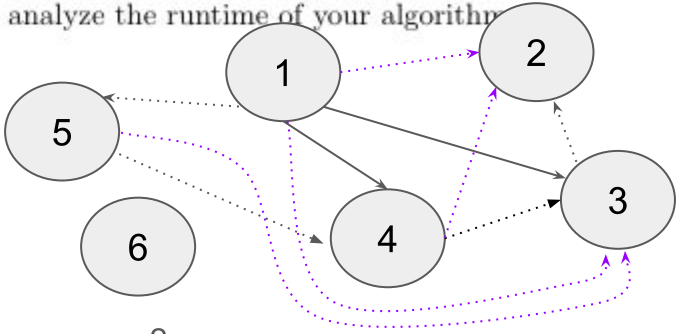


G

G^T



G^2

Vertex	Adj
1	3,**4**,5
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4
2	
3	
4	
5	
6	

Idea: when adding edge (i,j), **Add all edges pointing to i to j**

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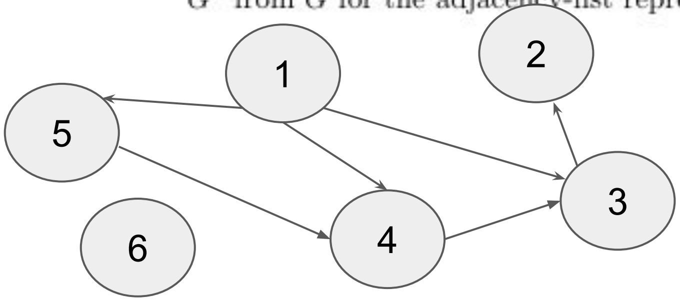
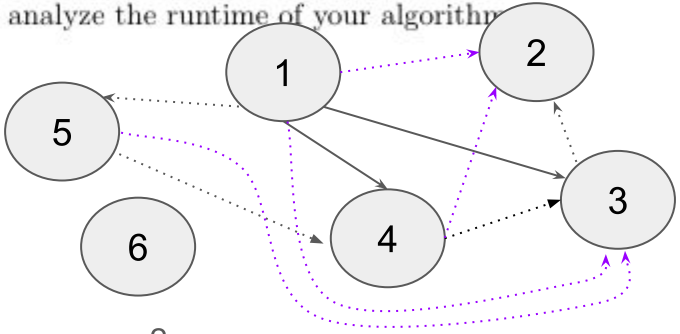


G

G^T



G^2

Vertex	Adj
1	3,**4**,5
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4
2	
3	
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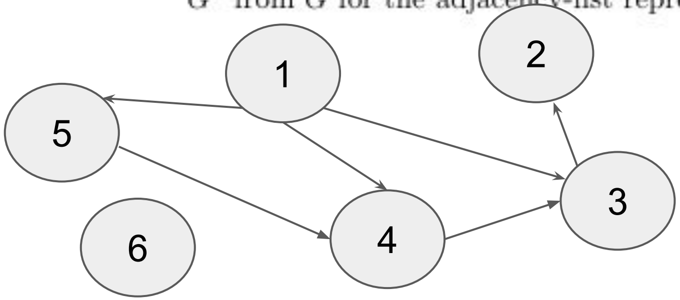
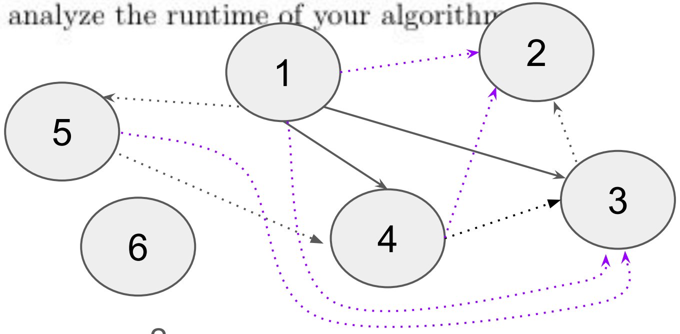


G

G^T



G^2

Vertex	Adj
1	3,4,**5**
2	
3	2
4	3
5	4
6	

Vertex	Adj
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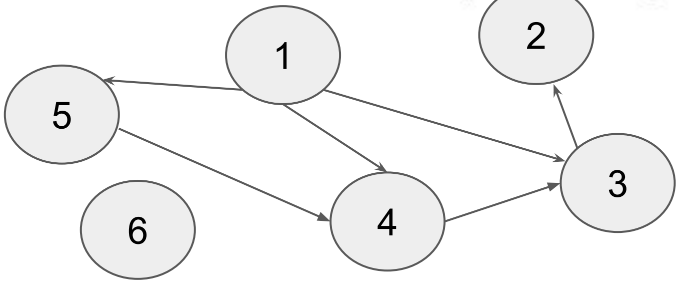
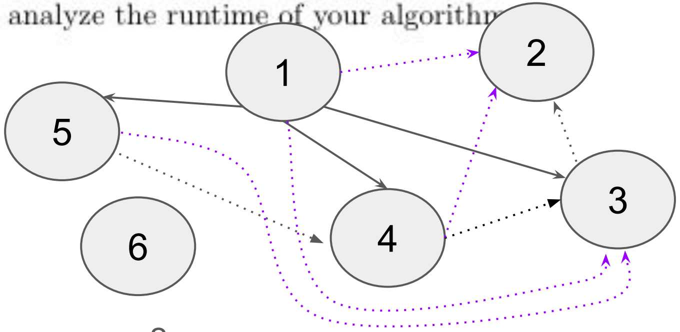


G

G^T



G^2

Vertex	Adj
1	3,4,**5**
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
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3	1 4
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5	1
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1	3,4,5
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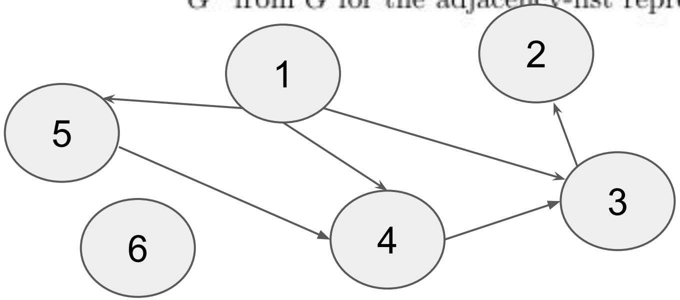
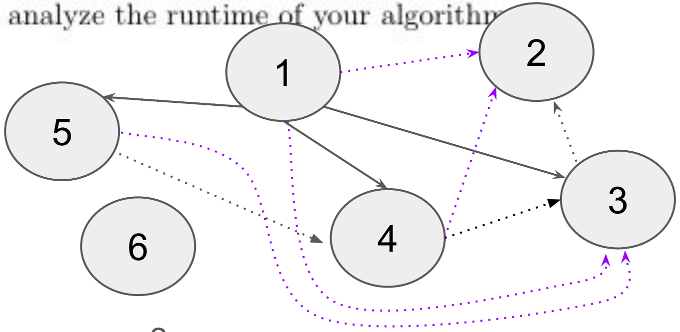


G

G^T



G^2

Vertex	Adj
1	3,4,**5**
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5
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3	
4	
5	
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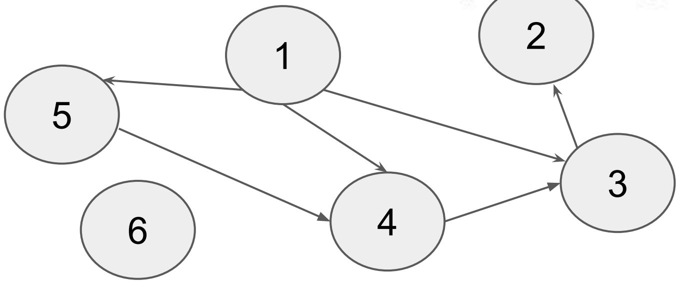

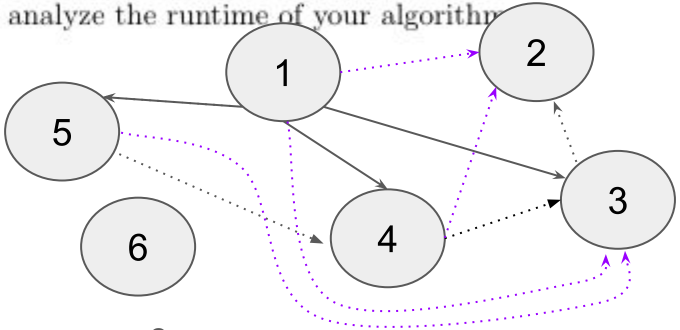

G $\qquad$ G^T $\qquad$ G^2

Vertex	Adj
1	3,4,5
2	
3	**2**
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5
2	
3	
4	
5	
6	

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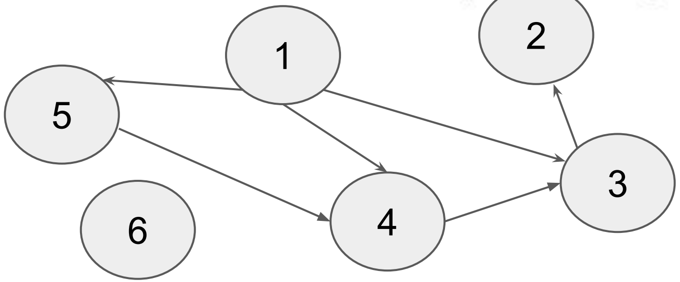
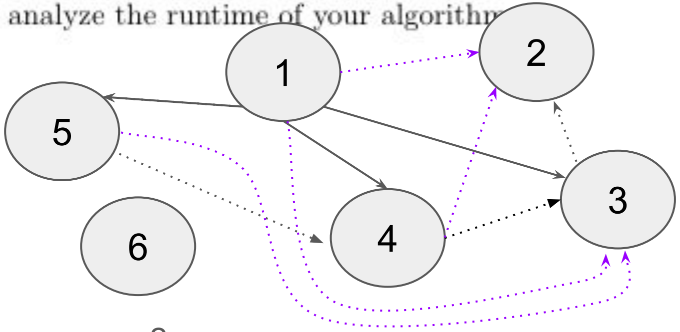


G

G^T



G^2

Vertex	Adj
1	3,4,5
2	
3	**2**
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
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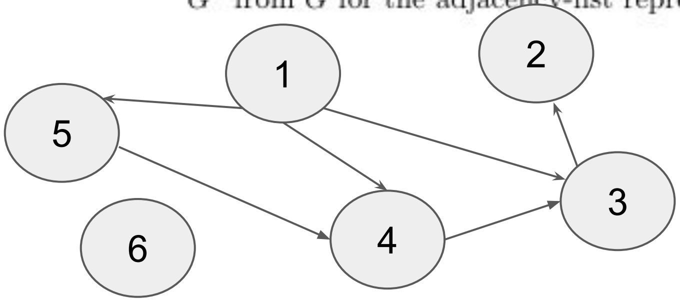

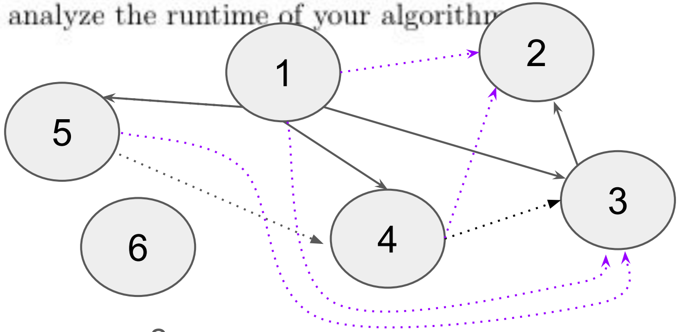

G $\quad$ G^T $\quad$ G^2

Vertex	Adj
1	3,4,5
2	
3	**2**
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5
2	
3	
4	
5	
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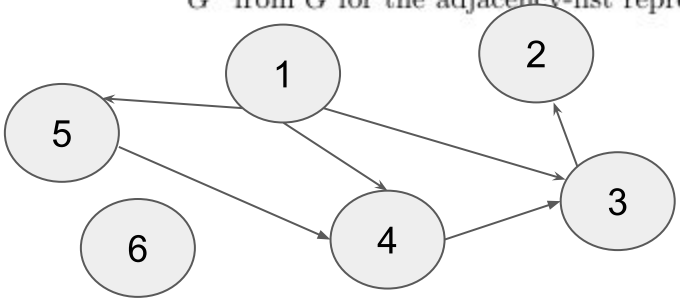

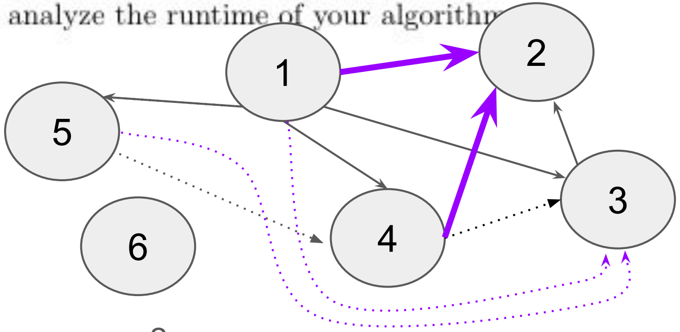

G

G^T

G^2

Vertex	Adj
1	3,4,5
2	
3	**2**
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	**1 4**
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5,**2**
2	
3	
4	**2**
5	
6	

Idea: when adding edge (i,j), **Add all edges pointing to i to j**

```
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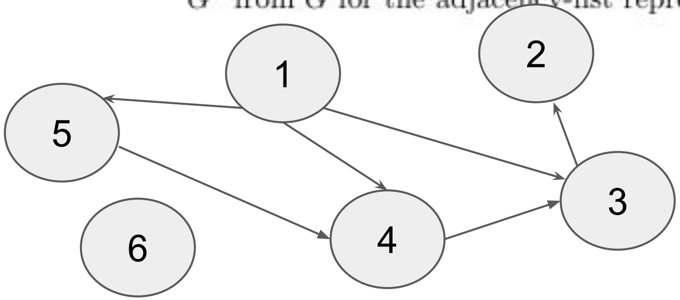
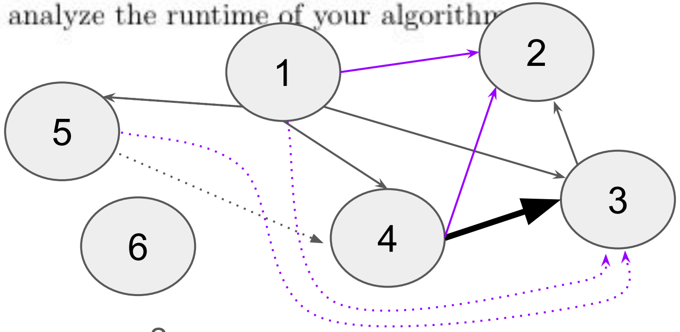


G

G^T



G^2

Vertex	Adj
1	3,4,5
2	
3	2
4	**3**
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5,2
2	
3	
4	2,3
5	
6	

Idea: when adding edge (i,j), **Add all edges pointing to i to j**

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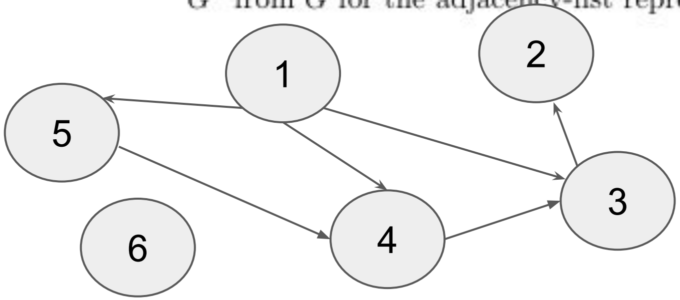
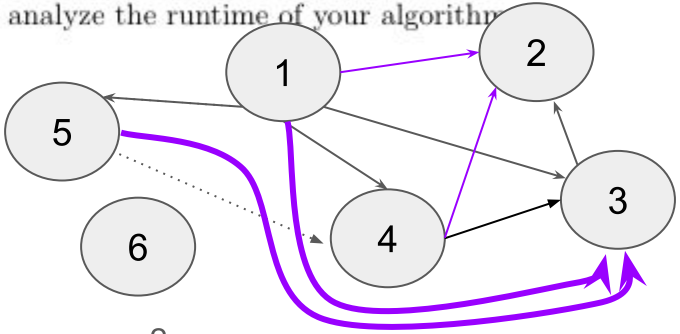


G

G^T



G^2

Vertex	Adj
1	3,4,5
2	
3	2
4	**3**
5	4
6	

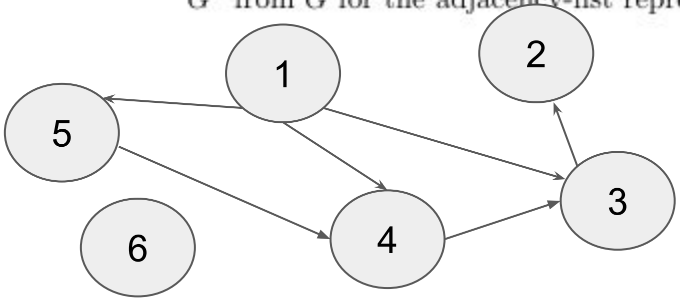
Vertex	Adj
1	
2	3
3	1 4
4	**1 5**
5	1
6	

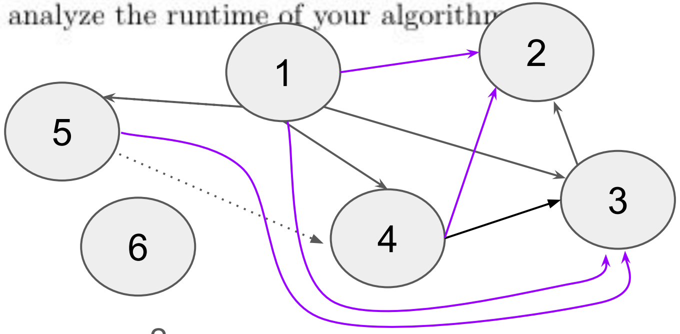
Vertex	Adj
1	3,4,5,2,**3**
2	
3	
4	2,3
5	**3**
6	

Idea: when adding edge (i,j), **Add all edges pointing to i to j**

For i in |V|:
    For j in G.adjList(i):
        add j to G^2.adjList(i)
        for k in G^T.adjList(i):
            add j to G^2.adjList(k)

3. The square of a directed graph $G = (V, E)$ is the graph $G^2 = (V, E^2)$, where $(u, v) \in E^2$ if and only if G contains a path with at most two edges between u and v. Describe an efficient algorithm for computing G^2 from G for the adjacency-list representations of G and analyze the runtime of your algorithm.





G $\qquad$ G^T $\qquad$ G^2

Vertex	Adj
1	3,4,5
2	
3	2
4	3
5	**4**
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5,2,3
2	
3	
4	2,3
5	3
6	

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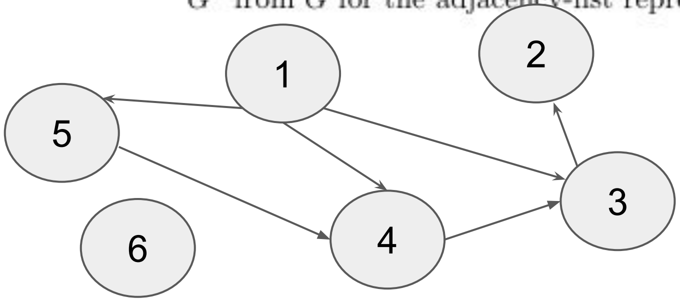

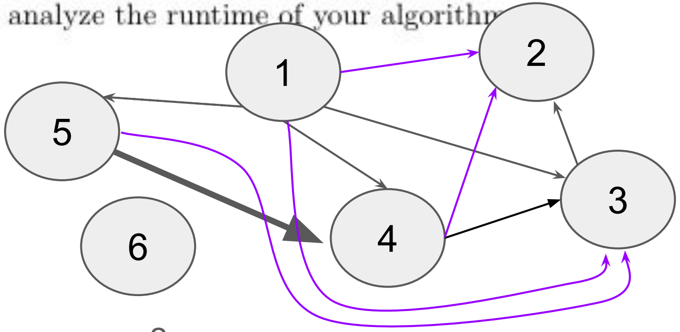

G $\qquad$ G^T $\qquad$ G^2

Vertex	Adj
1	3,4,5
2	
3	2
4	3
5	**4**
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5,2,3
2	
3	
4	2,3
5	3,4
6	

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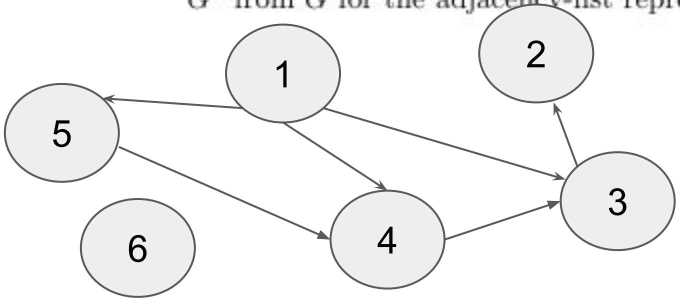
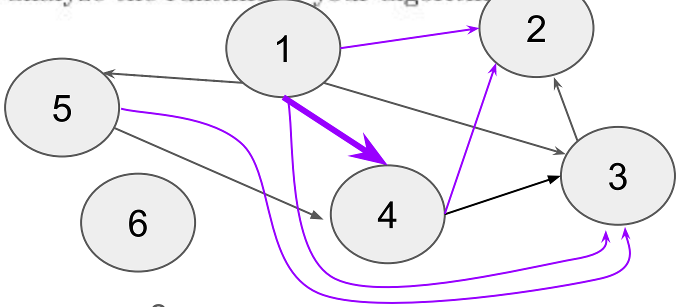


G

G^T



G^2

Vertex	Adj
1	3,4,5
2	
3	2
4	3
5	**4**
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	**1**
6	

Vertex	Adj
1	3,4,5,2,3,4
2	
3	
4	2,3
5	3,4
6	

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For i in |V|:
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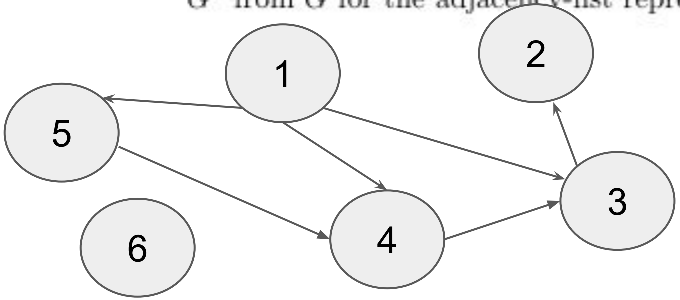
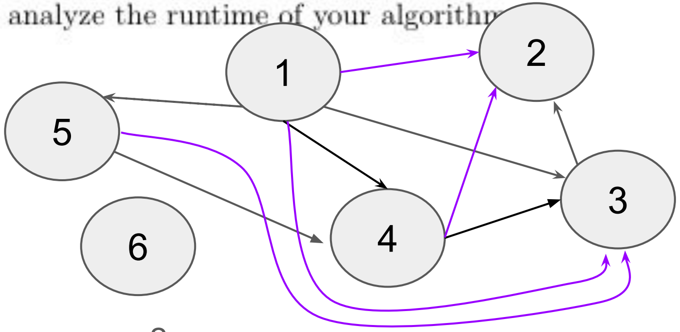


G

G^T



G^2

Vertex	Adj
1	3,4,5
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5,2,3, 4
2	
3	
4	2,3
5	3,4
6	

Idea: when adding edge (i,j), **Add all edges pointing to i to j**

```
For i in |V|:
        For j in G.adjList(i):
                add j to G^2.adjList(i)
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```

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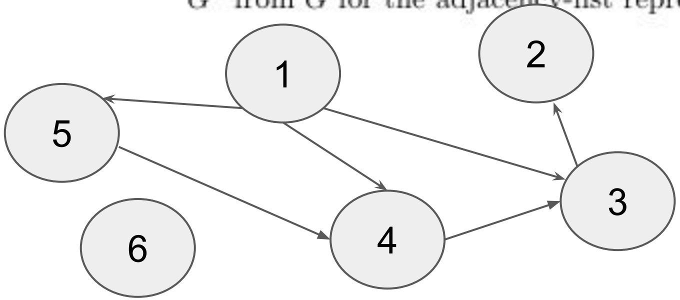
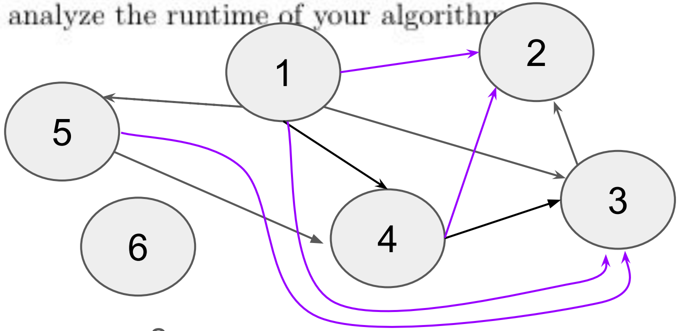


G

G^T



G^2

Vertex	Adj
1	3,4,5
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5,2,3,4
2	
3	
4	2,3
5	3,4
6	

Idea: when adding edge (i,j), **Add all edges pointing to i to j**

For i in |V|:
    For j in G.adjList(i):
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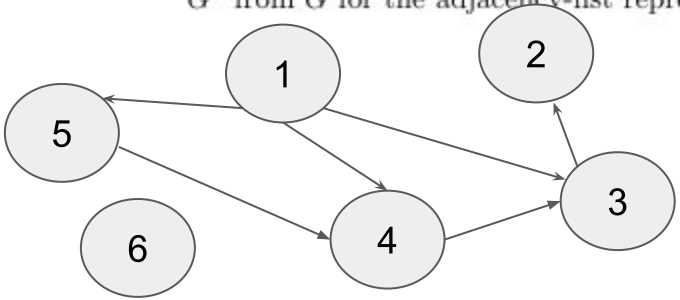
        for k in G^T.adjList(i):
            add j to G^2.adjList(k)

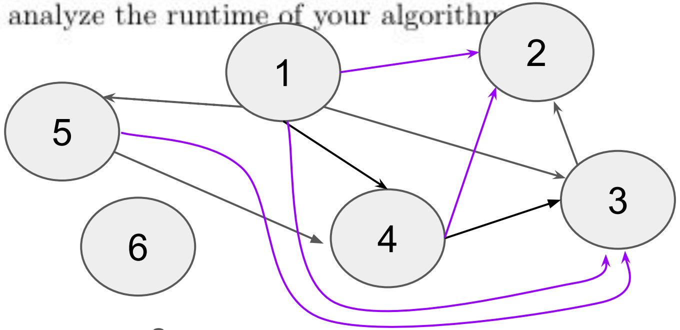
Time complexity?

3. The square of a directed graph $G = (V, E)$ is the graph $G^2 = (V, E^2)$, where $(u, v) \in E^2$ if and only if G contains a path with at most two edges between u and v. Describe an efficient algorithm for computing G^2 from G for the adjacency-list representations of G and analyze the runtime of your algorithm.



G

G^T



G^2

Vertex	Adj
1	3,4,5
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5,2,3,4
2	
3	
4	2,3
5	3,4
6	

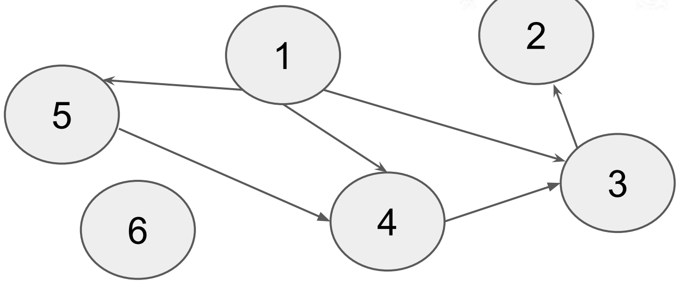
Idea: when adding edge (i,j), **Add all edges pointing to i to j**

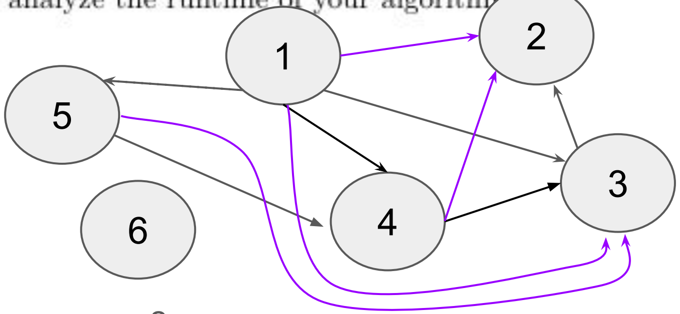
```
For i in |V|:
        For j in G.adjList(i):
                add j to G^2.adjList(i)
                for k in G^T.adjList(i):
                        add j to
                          G^2.adjList(k)
```

Time complexity?

Time to process each edge in G = Look through an adj list <= |V|

3. The square of a directed graph $G = (V, E)$ is the graph $G^2 = (V, E^2)$, where $(u, v) \in E^2$ if and only if G contains a path with at most two edges between u and v. Describe an efficient algorithm for computing G^2 from G for the adjacency-list representations of G and analyze the runtime of your algorithm.





G $\qquad$ G^T $\qquad$ G^2

Vertex	Adj
1	3,4,5
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5,2,3, 4
2	
3	
4	2,3
5	3,4

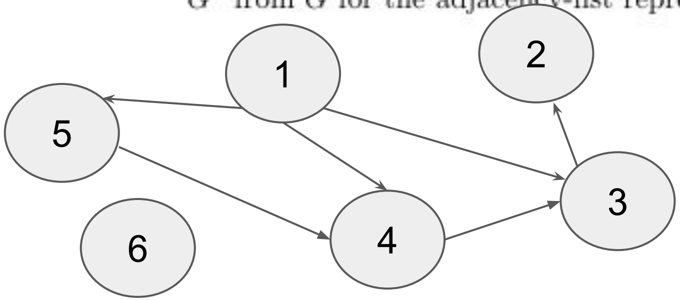
Idea: when adding edge (i,j), **Add all edges pointing to i to j**

```
For i in |V|:
    For j in G.adjList(i):
        add j to G^2.adjList(i)
        for k in G^T.adjList(i):
            add j to
              G^2.adjList(k)
```

Time complexity?

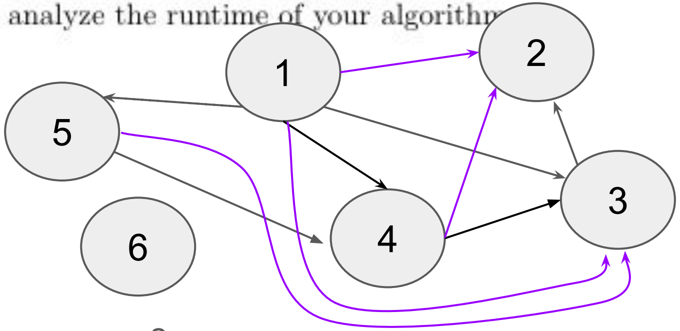
Time to process each edge in G = O(|V|)

3. The square of a directed graph $G = (V, E)$ is the graph $G^2 = (V, E^2)$, where $(u, v) \in E^2$ if and only if G contains a path with at most two edges between u and v. Describe an efficient algorithm for computing G^2 from G for the adjacency-list representations of G and analyze the runtime of your algorithm.



G

G^T



G^2

Vertex	Adj
1	3,4,5
2	
3	2
4	3
5	4
6	

Vertex	Adj
1	
2	3
3	1 4
4	1 5
5	1
6	

Vertex	Adj
1	3,4,5,2,3, 4
2	
3	
4	2,3
5	3,4
6	

Idea: when adding edge (i,j), **Add all edges pointing to i to j**

```
For i in |V|:
    For j in G.adjList(i):
        add j to G².adjList(i)
        for k in Gᵀ.adjList(i):
            add j to
             G².adjList(k)
```

Time complexity?

O(|E||V|)

Question 2

(Adjacency-matrix Representation)

1. Give an adjacency-matrix representation for a complete binary search tree on 7 vertices numbered from 1 to 7.

2. Show how to determine in $O(|V|)$ time, whether a directed graph G contains a **universal-sink**, i.e. a vertex with in-degree $|V| - 1$ and out-degree 0, given an adjacency-matrix for G.

What in the world in an adjacency matrix?

Question 2

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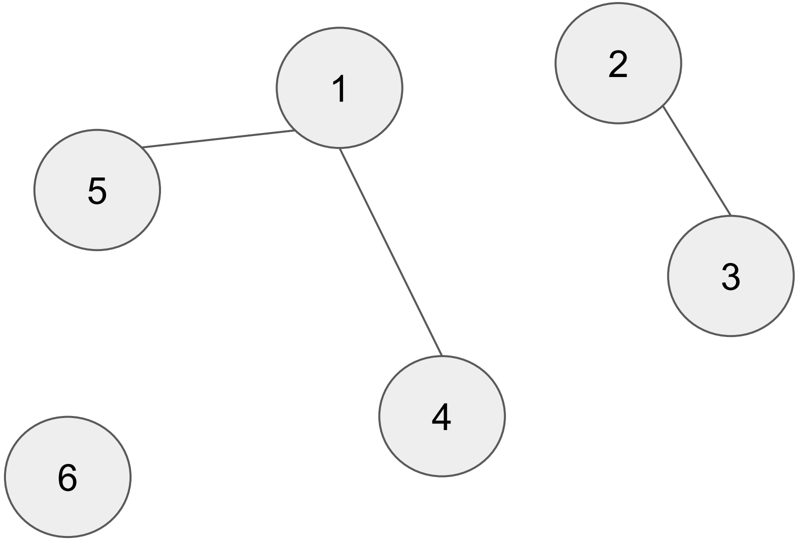
What in the world in an adjacency matrix?

Adjacency Matrix

Edges represented in a |V| x |V| matrix

E.g. if undirected..



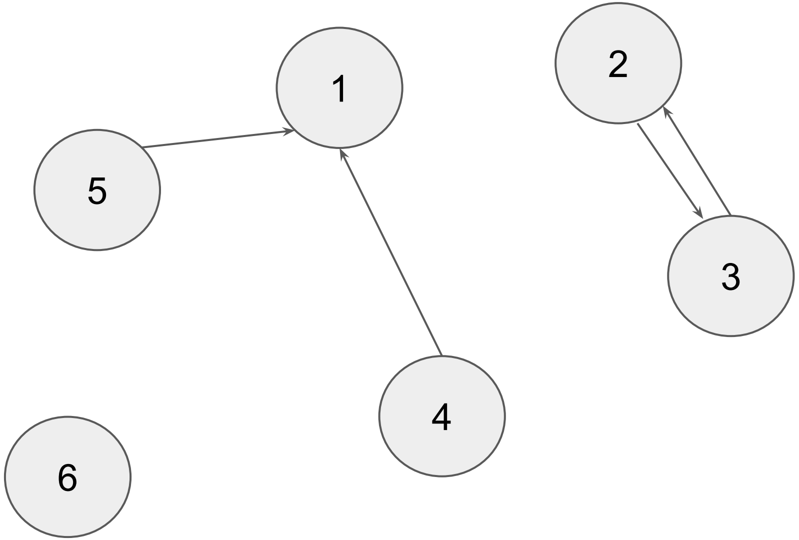
$$\begin{array}{cc} & \begin{matrix}1 & 2 & 3 & 4 & 5 & 6\end{matrix} \\ \begin{matrix}1\\2\\3\\4\\5\\6\end{matrix} & \left(\begin{matrix} \; & \; & \; & \; & \; & \; \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \\ & & & & & \end{matrix}\right)\end{array}$$

Adjacency Matrix

“Row goes to column”

Edges represented in a |V| x |V| matrix

E.g. if **directed**..



1 2 3 4 5 6

1
2
3
4
5
6

Question 2

(Adjacency-matrix Representation)

1. Give an adjacency-matrix representation for a complete binary search tree on 7 vertices numbered from 1 to 7.

2. Show how to determine in $O(|V|)$ time, whether a directed graph G contains a **universal-sink**, i.e. a vertex with in-degree $|V| - 1$ and out-degree 0, given an adjacency-matrix for G.

Someone give me a complete binary search tree

Question 2

(Adjacency-matrix Representation)

1. Give an adjacency-matrix representation for a complete binary search tree on 7 vertices numbered from 1 to 7.

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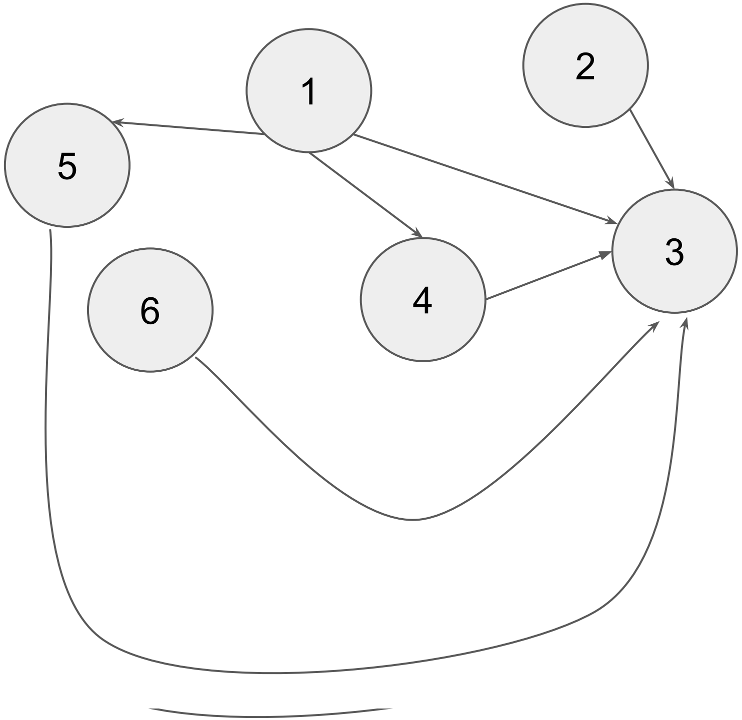
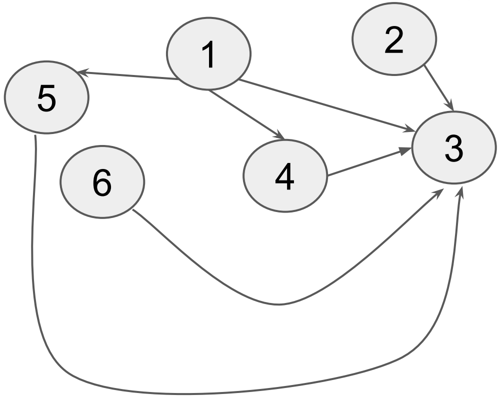


	1	2	3	4	5	6
1	0	0	1	1	1	0
2	0	0	1	0	0	0
3	0	0	0	0	0	0
4	0	0	1	0	0	0
5	0	0	1	0	0	0
6	0	0	1	0	0	0

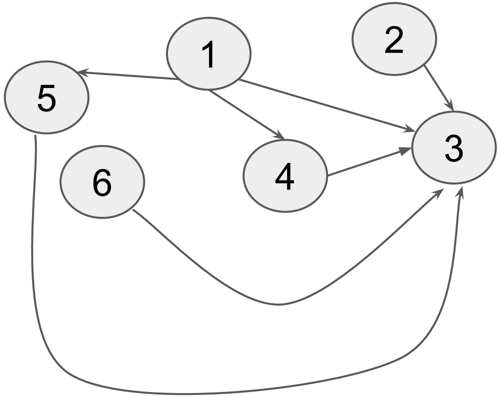
2. Show how to determine in $O(|V|)$ time, whether a directed graph G contains a **universal-sink**, i.e. a vertex with in-degree $|V|-1$ and out-degree 0, given an adjacency-matrix for G.



What do I notice about the adjacency matrix (specifically column 3)

$$
\begin{array}{c|cccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & 0 & 0 & 1 & 1 & 1 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 1 & 0 & 0 & 0 \\
5 & 0 & 0 & 1 & 0 & 0 & 0 \\
6 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}
$$

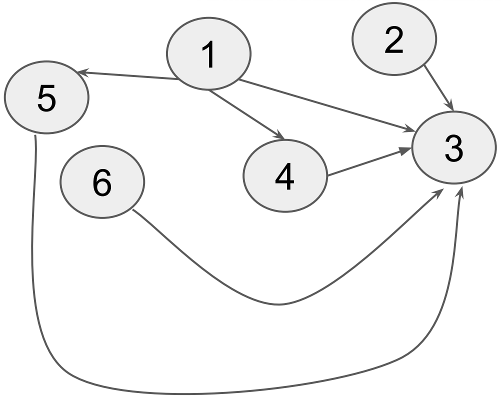
2. Show how to determine in $O(|V|)$ time, whether a directed graph G contains a **universal-sink**, i.e. a vertex with in-degree $|V|-1$ and out-degree 0, given an adjacency-matrix for G.



Obs 1: If universal sink, col 3 has 1 in every entry except (3,3)

$$\begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 5 & 0 & 0 & 1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}$$

2. Show how to determine in $O(|V|)$ time, whether a directed graph G contains a **universal-sink**, i.e. a vertex with in-degree $|V|-1$ and out-degree 0, given an adjacency-matrix for G.



Obs 1: If universal sink, col 3 has 1 in every entry except (3,3)

Obs 2: row 3 is all 0s

	1	2	3	4	5	6
1	0	0	1	1	1	0
2	0	0	1	0	0	0
3	0	0	0	0	0	0
4	0	0	1	0	0	0
5	0	0	1	0	0	0
6	0	0	1	0	0	0

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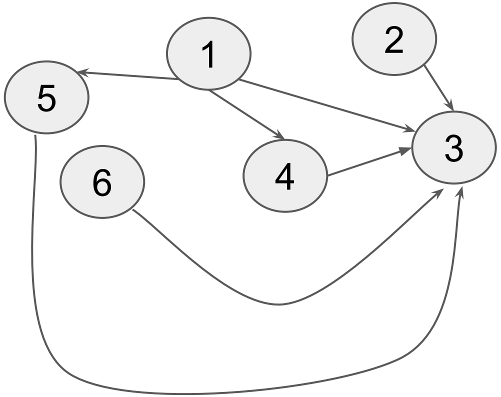


$$
\begin{array}{c|cccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & 0 & 0 & 1 & 1 & 1 & 0 \\
2 & 0 & 0 & 1 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 1 & 0 & 0 & 0 \\
5 & 0 & 0 & 1 & 0 & 0 & 0 \\
6 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}
$$

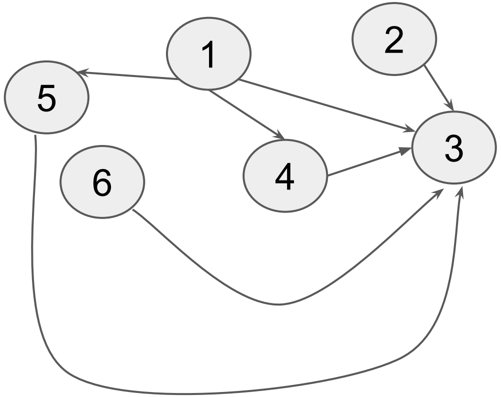
Obs 1: If universal sink, col 3 has 1 in every entry except (3,3)

Obs 2: row 3 is all 0s

Algorithm:

1. Start at (i,j) = (1,1) entry

2. Show how to determine in $O(|V|)$ time, whether a directed graph G contains a **universal-sink**, i.e. a vertex with in-degree $|V|-1$ and out-degree 0, given an adjacency-matrix for G.



$$
\begin{array}{c|cccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & 0 & 0 & 1 & 1 & 1 & 0 \\
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3 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 1 & 0 & 0 & 0 \\
5 & 0 & 0 & 1 & 0 & 0 & 0 \\
6 & 0 & 0 & 1 & 0 & 0 & 0
\end{array}
$$

Obs 1: If universal sink, col 3 has 1 in every entry except (3,3)

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Algorithm:

1. Start at (i,j) = (1,1) entry
2. while i < |V| :

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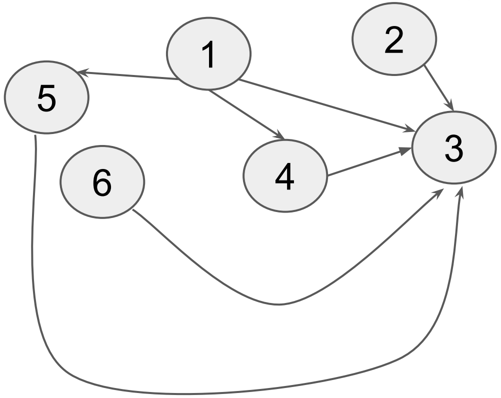


	1	2	3	4	5	6
1	0	0	1	1	1	0
2	0	0	1	0	0	0
3	0	0	0	0	0	0
4	0	0	1	0	0	0
5	0	0	1	0	0	0
6	0	0	1	0	0	0

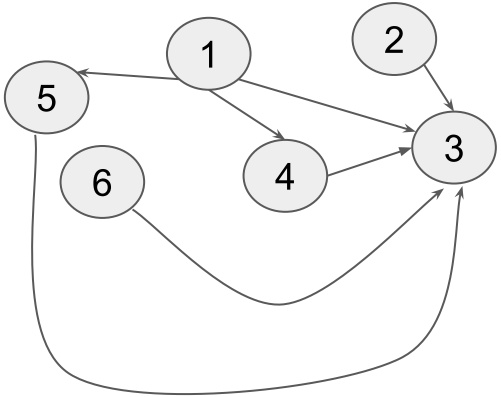
Obs 1: If universal sink, col 3 has 1 in every entry except (3,3)

Obs 2: row 3 is all 0s

Algorithm:

1. Start at (i,j) = (1,1) entry
2. while i < |V| :
   a. If entry(i,j) = 0: i += 1 \\go right

2. Show how to determine in $O(|V|)$ time, whether a directed graph G contains a **universal-sink**, i.e. a vertex with in-degree $|V|-1$ and out-degree 0, given an adjacency-matrix for G.



	1	2	3	4	5	6
1	0	0	1	1	1	0
2	0	0	1	0	0	0
3	0	0	0	0	0	0
4	0	0	1	0	0	0
5	0	0	1	0	0	0
6	0	0	1	0	0	0

Obs 1: If universal sink, col 3 has 1 in every entry except (3,3)

Obs 2: row 3 is all 0s

Algorithm:

1. Start at (i,j) = (1,1) entry
2. while i < |V| :
   a. If entry(i,j) = 0: i += 1 \\go right
   b. else: j += 1 \\go down

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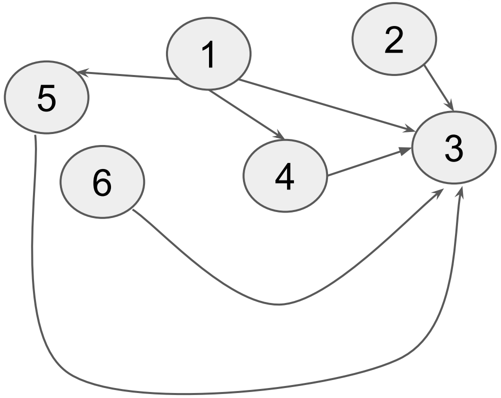

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	1	2	3	4	5	6
1	0	0	1	1	1	0
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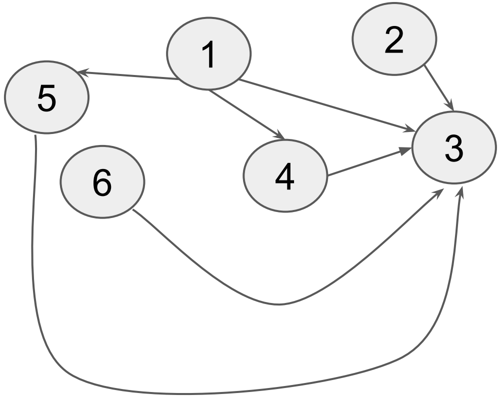

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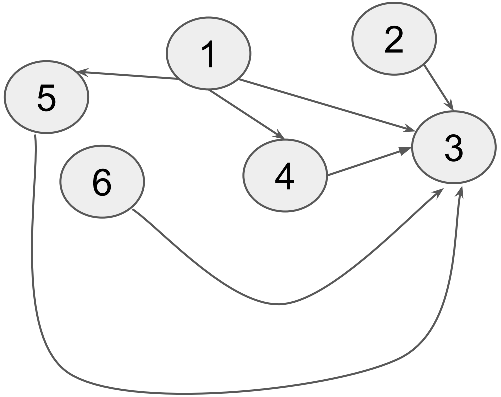

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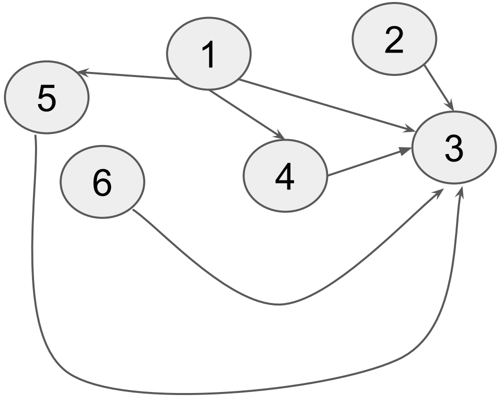

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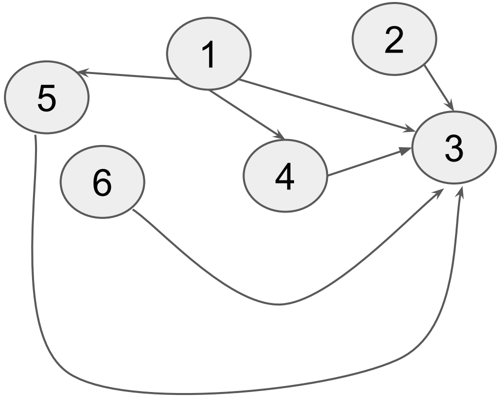

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Question 3

(Graphs)

1. True or False:

(1) There exists a simple, undirected graph with 5 nodes, each of degree 3.

(2) There exists a simple, undirected graph G with n vertices, whose vertex degrees are $0, 1, 2, \ldots, n-1$. (assume $n > 3$)

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We can try..

1. True or False:

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Hmm if I can’t come up with an example, prob false

The answer key says..

False. For an undirected graph, the total degree should be a even number. But $5 * 3 = 15$, which is odd.

But I don’t know what this means lol

Why should the degree be an even number?

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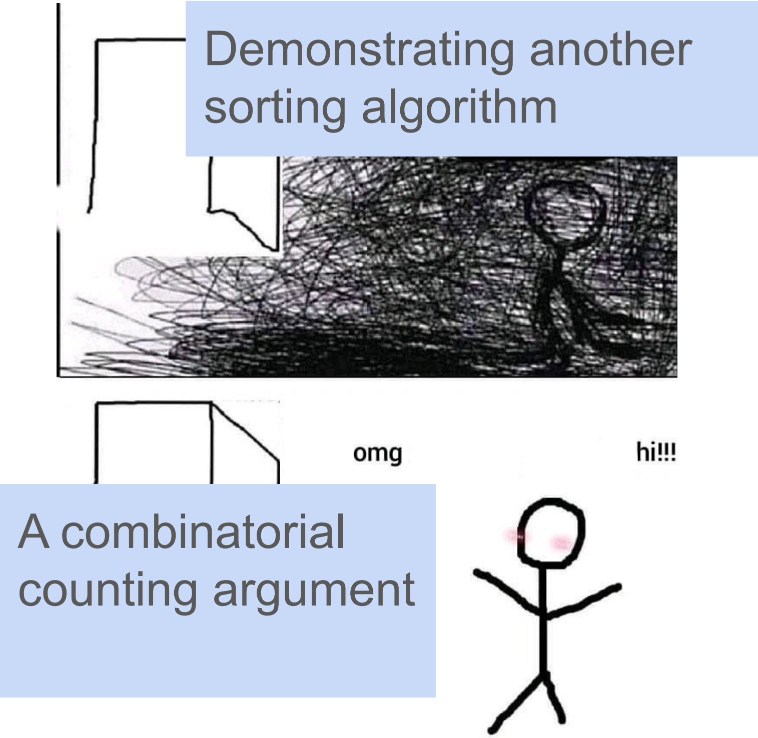

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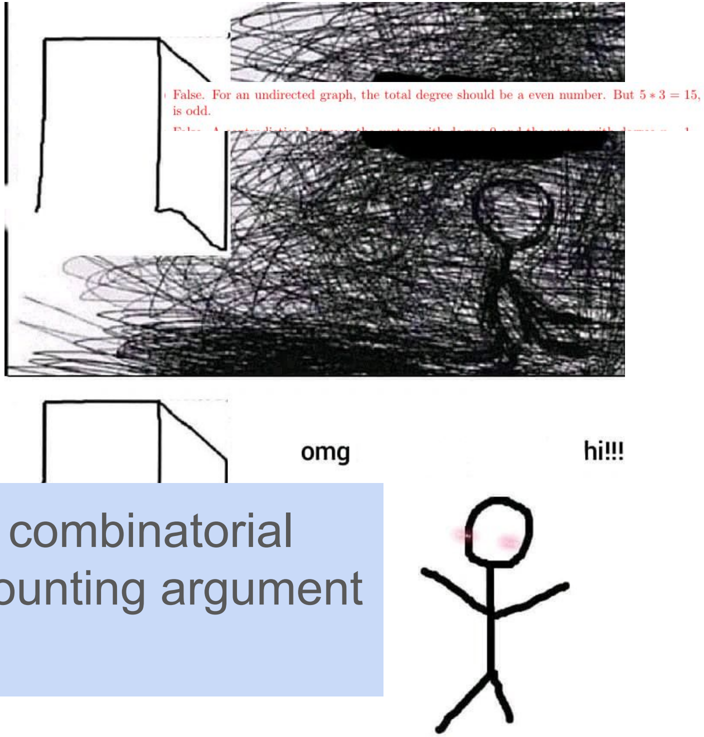

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But the maximum number of edges in a

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This is less than 15, contradiction!

A combinatorial counting argument

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We can try.. (again)

(2) There exists a simple, undirected graph G with n vertices, whose vertex degrees are $0, 1, 2, \ldots, n-1$. (assume $n > 3$)

We can't do this because a vertex with degree n - 1 connects to all other vertices

There cannot be a vertex with 0 degree

Easy peasy

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Side note: I would say this is a theorem

Lemmas: intermediate results used to prove theorems
Corollaries: easy follow-ups to to theorems

IMO this stands on its own

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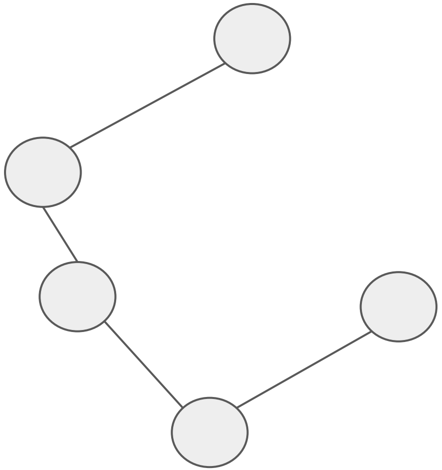
WTS: 1 + 2 -> 3

Suppose G is connected and acyclic



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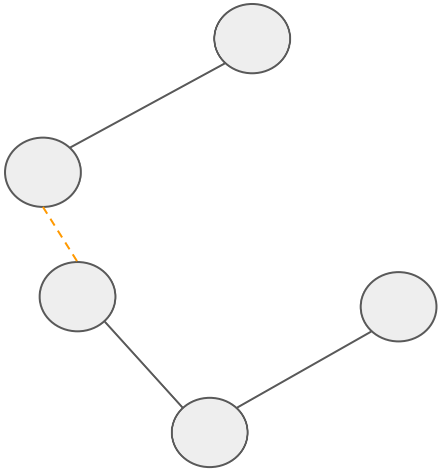
Can $|E| < |V| - 1$?



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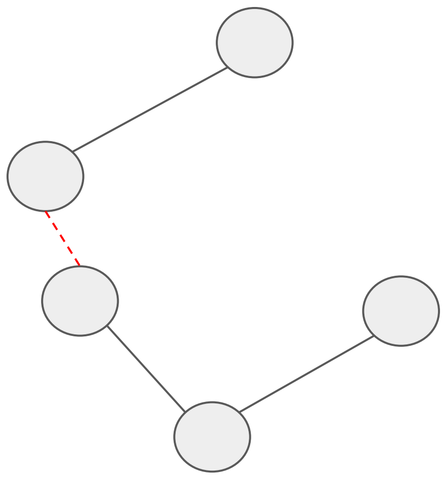
Intuition: lose connectivity

Lets prove this



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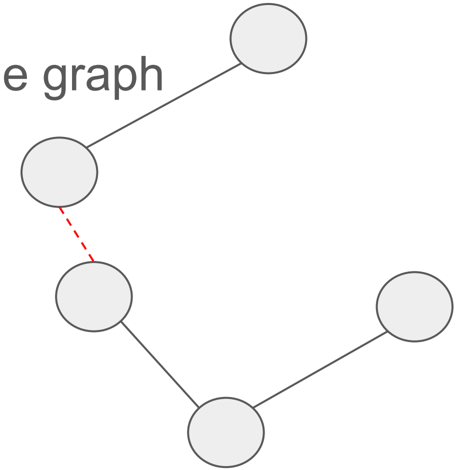
Connectivity implies I can take a walk to every vertex on the graph



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Say I talk this walk starting on some vertex,

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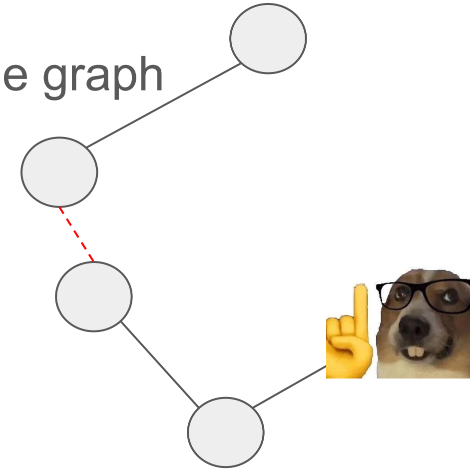

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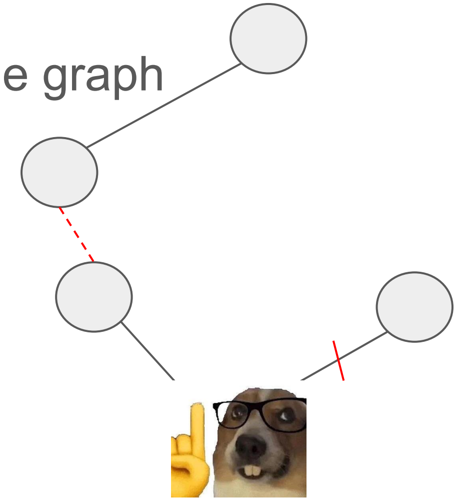

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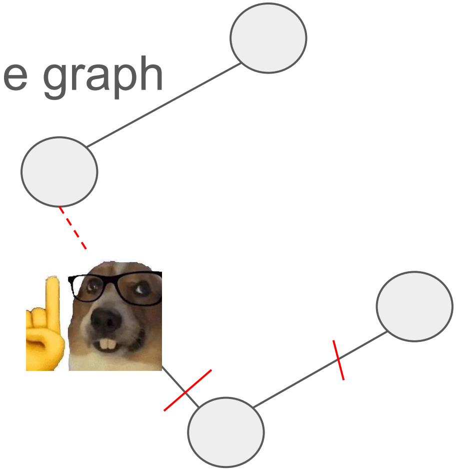

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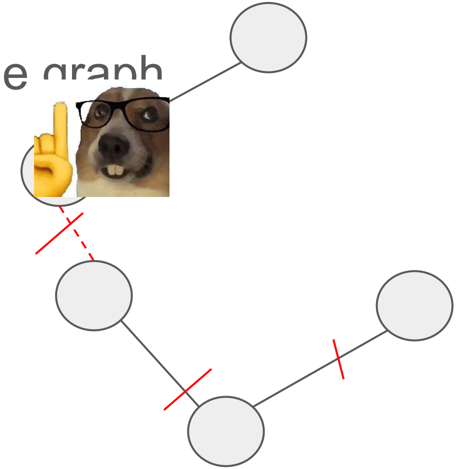

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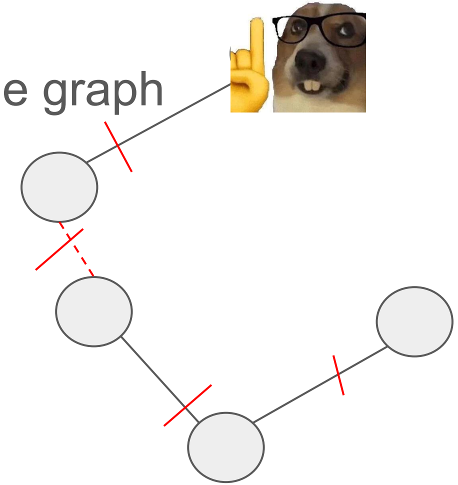

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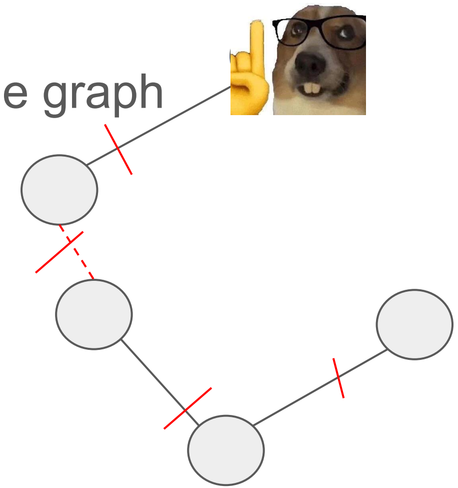

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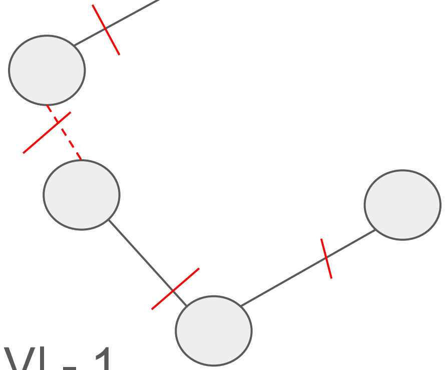

For each unique vertex I visit, I had to take an edge there

Since I visit |V| - 1 unique vertices (minus the start), |E| >= |V| - 1

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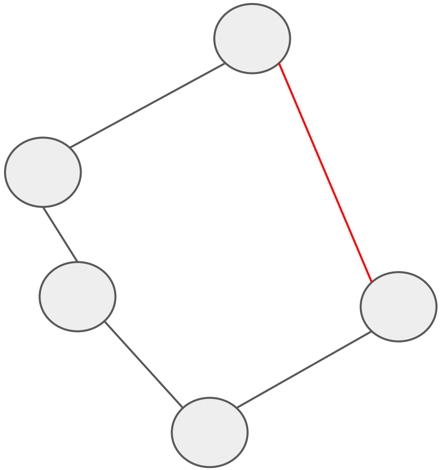
We showed |E| >= |V| - 1

Can |E| > |V| - 1?



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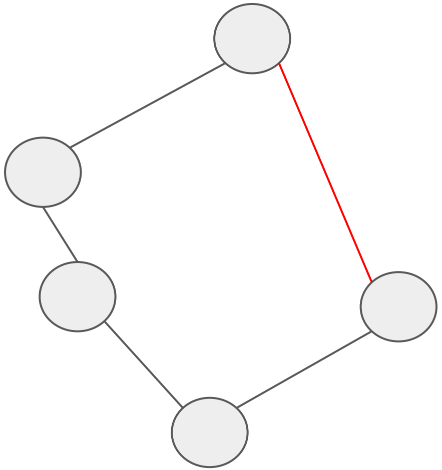

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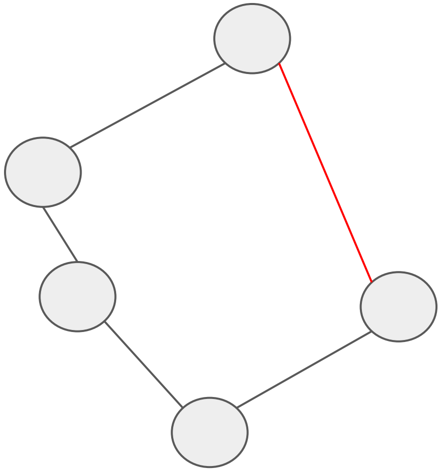

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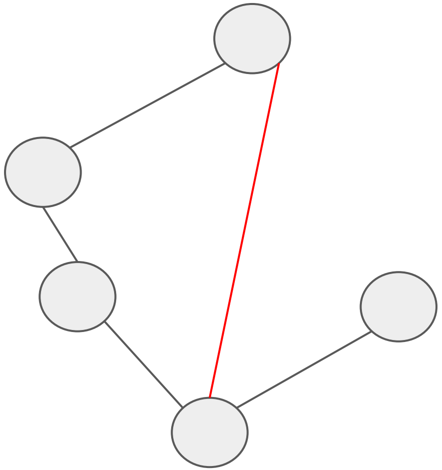

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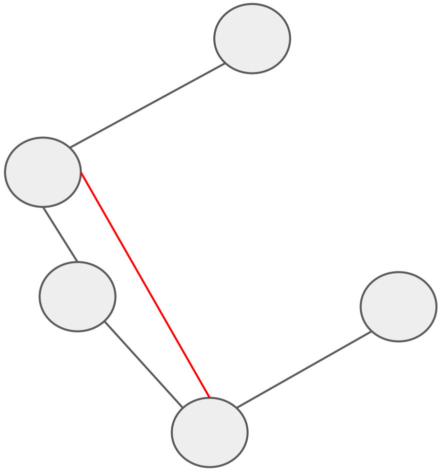

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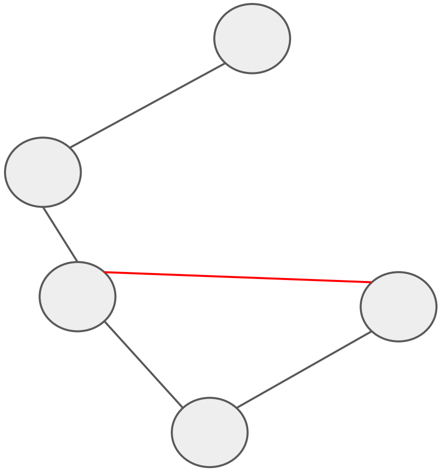
Assume for the sake of contradiction $|E| > |V| - 1$

(Anywhere I add the edge will create a cycle)

We argue this formally



$|V| = 5$, $|E| = 5$

2. A tree is the most widely used special type of graph, in a sense that it is the minimal connected graph. Prove the following important lemma:

Let G be an undirected graph, any two of the following properties imply the third property, and that G is a tree.

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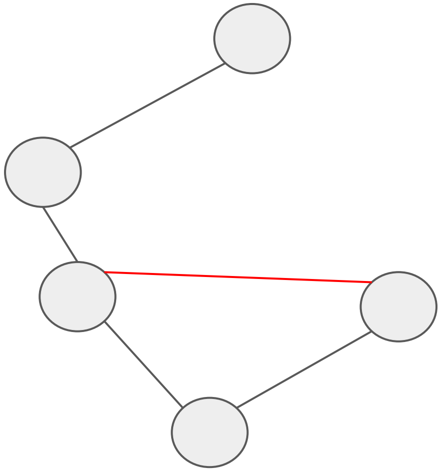

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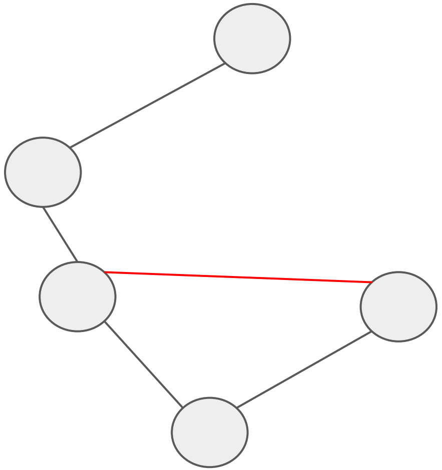

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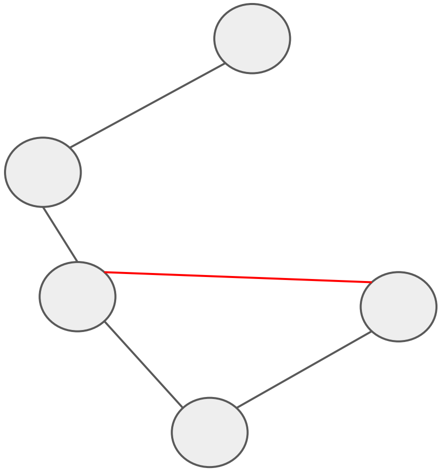
|V| nodes with |V| edges forms a cycle, contradiction!



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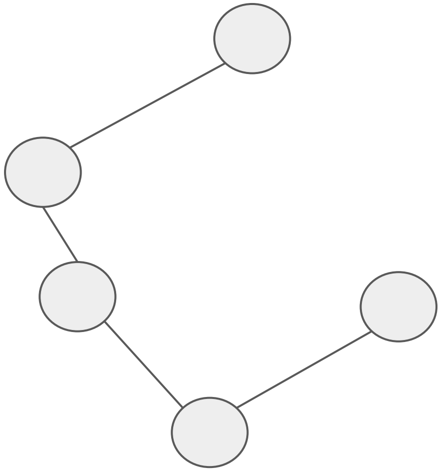
<u>WTS</u>: 1 + 3 -> 2

Suppose connected and |E| = |V| - 1

Can there be a cycle?



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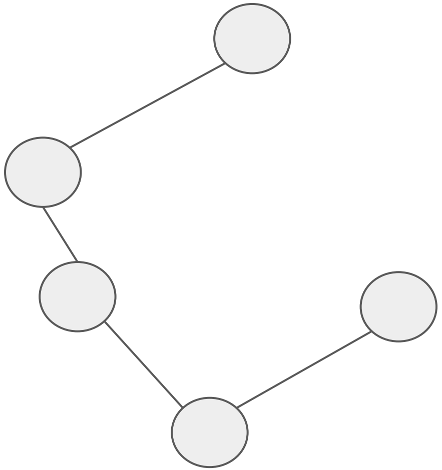

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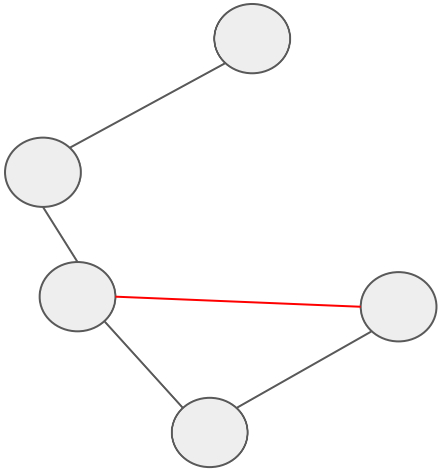

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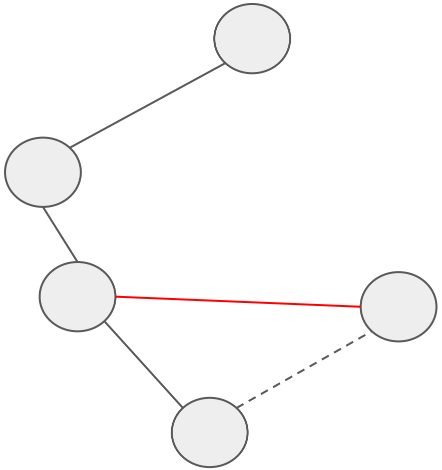

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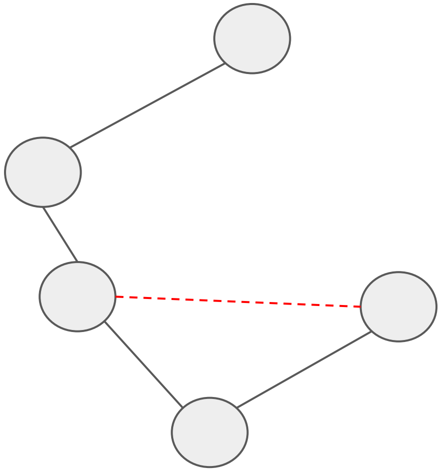
We still have connectivity with |V| -2 edges, contradiction



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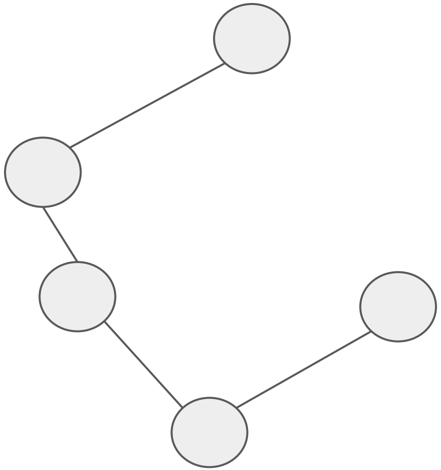

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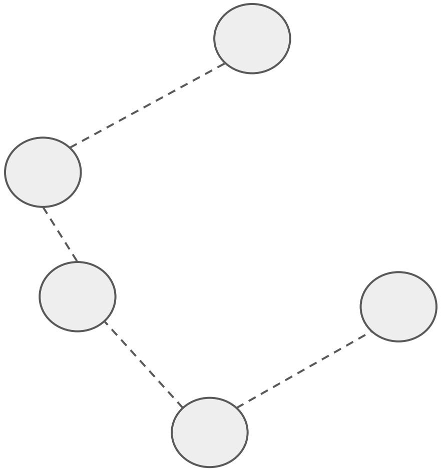

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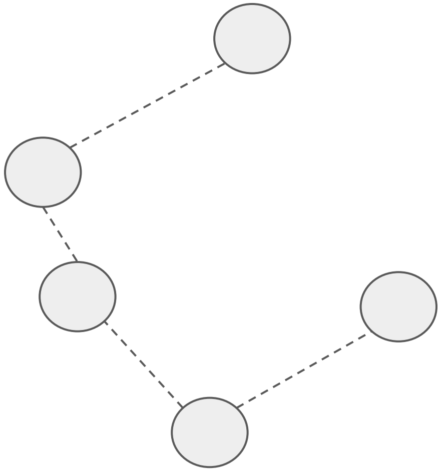

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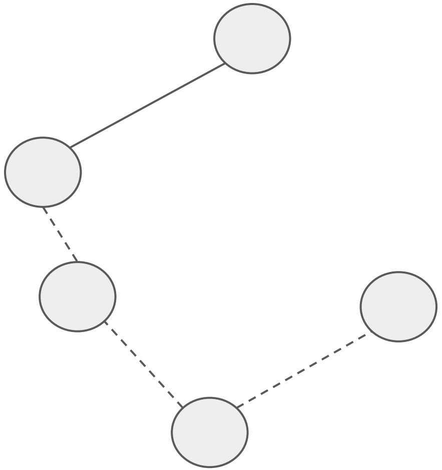

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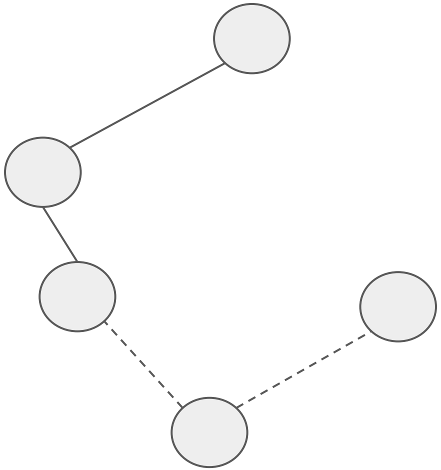
What do I notice?



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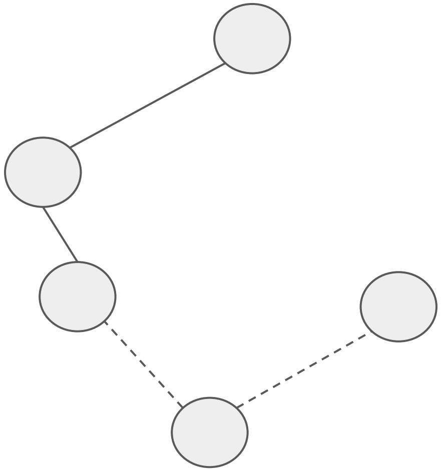

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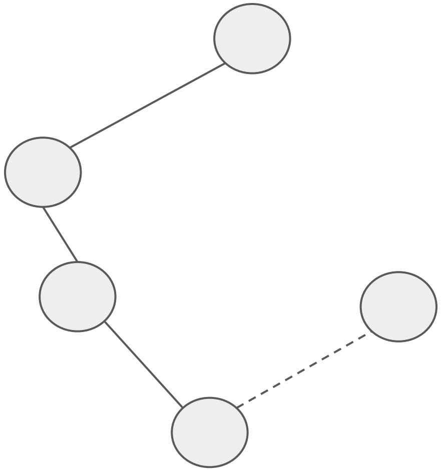

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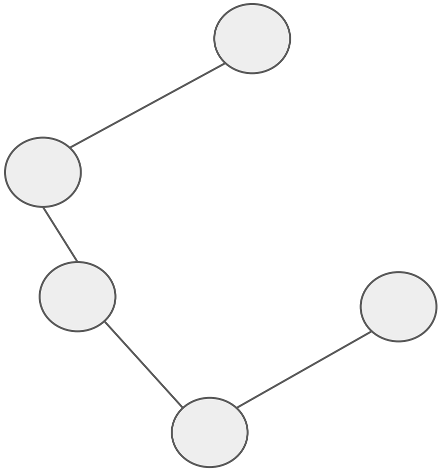
The edge I start with has 2 unique vertices



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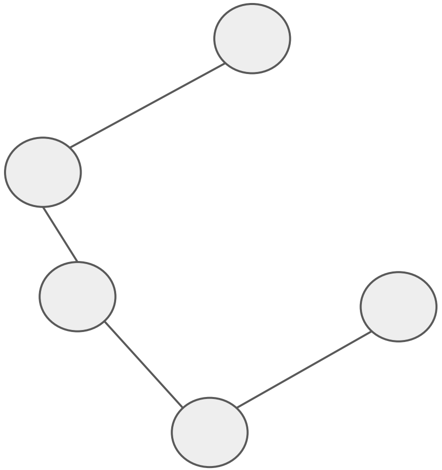

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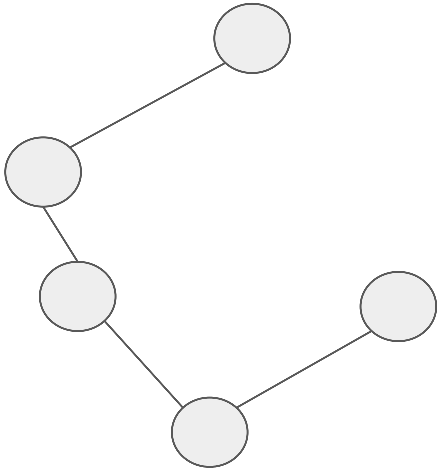


vertices seen = (2) + (|E| - 1) (1) = 2 + |V| - 2 = |V|, hence connected

|V| = 5, |E| = 4

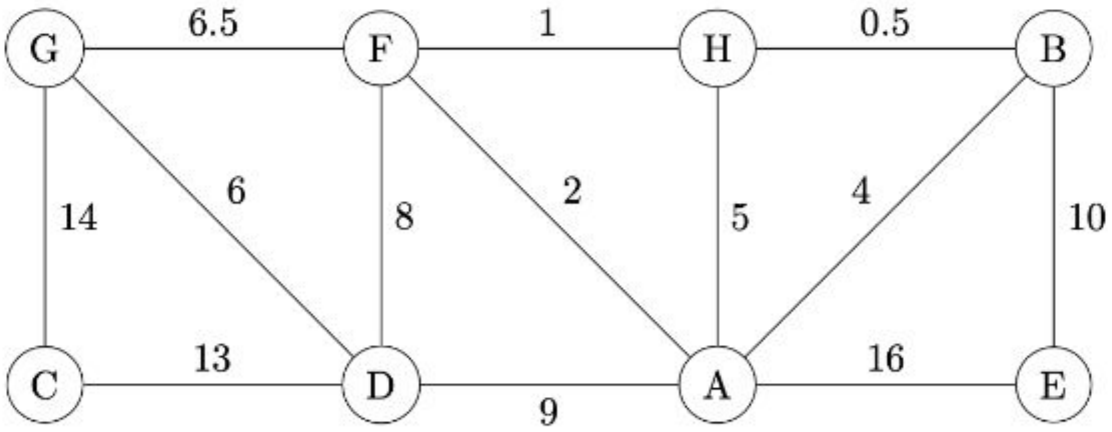
Question 4

(Review)

Consider the following undirected graph drawn below. For each part below we are only asking for the order in which edges are added. Assume that the graph is represented in adjacency-list form and that each adjacency-list is given in lexicographic order.

- List the order that edges are added to the BFS tree if we run BFS starting at node A.
- List the order that edges are added to the DFS tree if we run DFS starting at node A.

BFS



DFS

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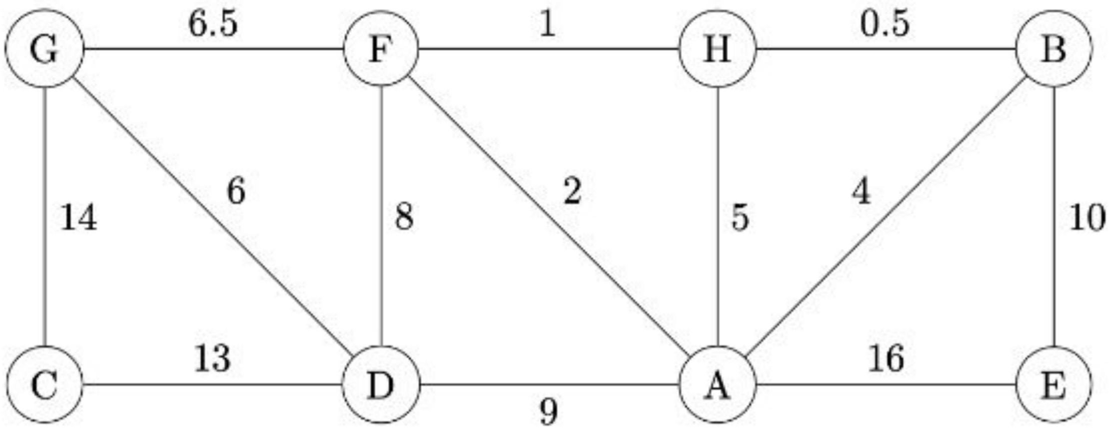
BFS



DFS

Question 5

(Breadth-first search)

1. What is the running time of BFS if we represent its input graph by an adjacency-matrix instead of the adjacency-list representation?

2. **(Diameter of a tree)** We know that the BFS finds the shortest path from the source s to each reachable vertex. Now let $T = (V, E)$ be a tree and define The *diameter* of a tree $\text{dia}(T)$ be the largest of all shortest-path distances in the tree. Think about how to use BFS to compute the diameter of a tree.

```
BFS(s):
    stack/queue(?) visit;
    Add s to visit
    while visit nonempty:
        v = visit.pop()
        If v not seen yet, add its neighbors to visit
```

Lets analyze cost

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Total cost: O(|V|^2)

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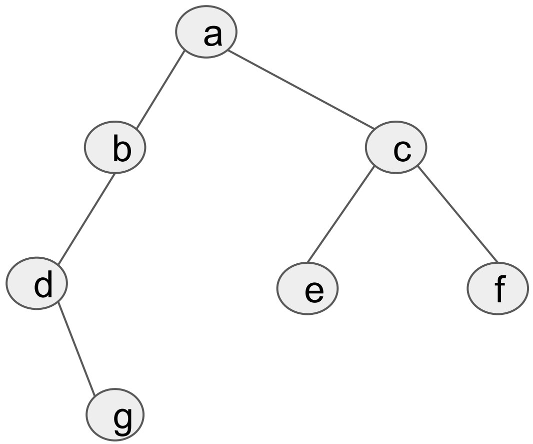

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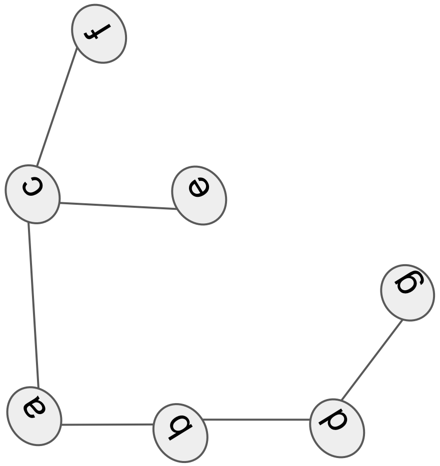

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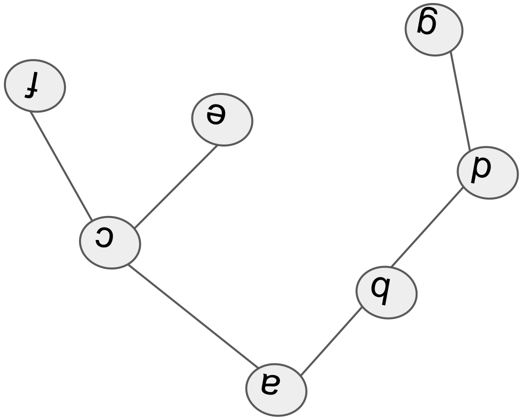

Fun fact: this is how trees look irl
(I touched grass 7 years ago)

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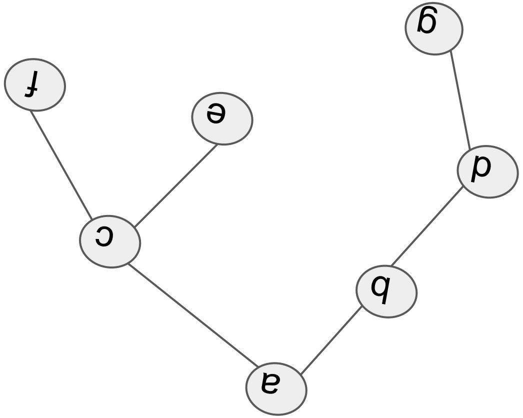
So the best we can do is just to run BFS from some starting node.



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Diameter = length of longest path in the tree

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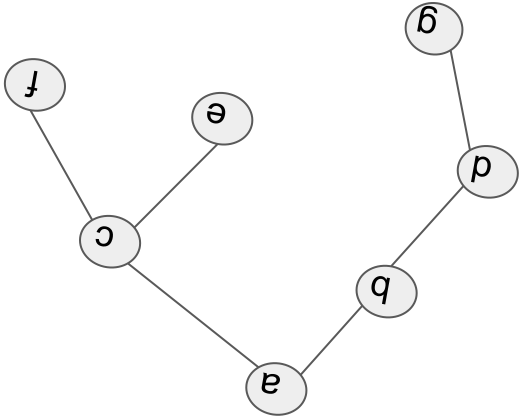

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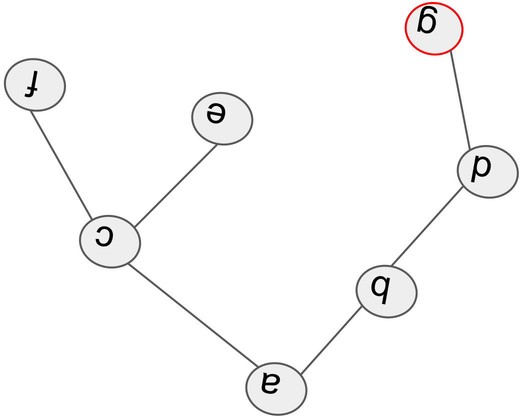

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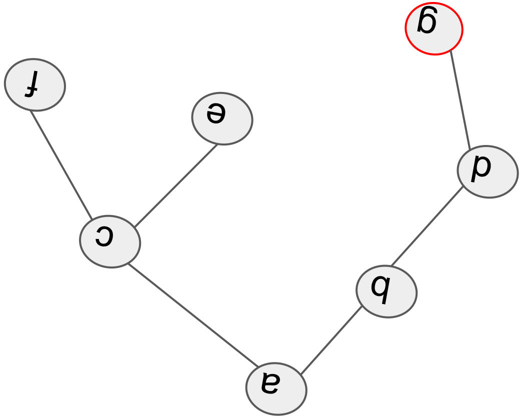

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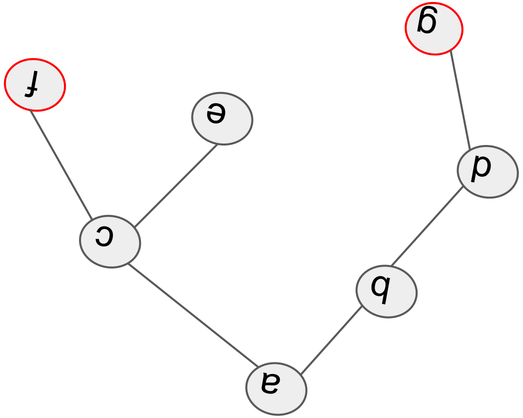

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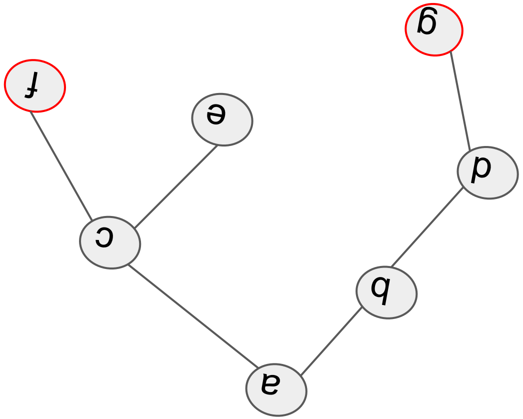

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Proof in the solution.pdf : kinda tedious but intuitively works

Question 6

(Depth-first search)

1. Is is possible that a vertex u of a directed graph G can end up in a depth-first tree containing only u, even though u has both incoming and outgoing edges in G?

2. **TRUE or False**: A directed graph G contains a path from u to v, and if u is visited before v in a DFS of G, then v must be a descendant of u in the corresponding DFS tree.

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HAVE A GREAT SPRING BREAK!