PSO 2

Question 1

(Recursion Tree) Give a big-O closed form for each of the following recurrences. (Assume that $T(x) = 1$ for any $x \leq 1$.)

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Warning: Solving this $T(n)$ using iterations is a bad idea!

… kind of, we will see that trees help us organize better!

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2. Find the cost at the ith level and the number of levels
3. Derive the sum and closed form

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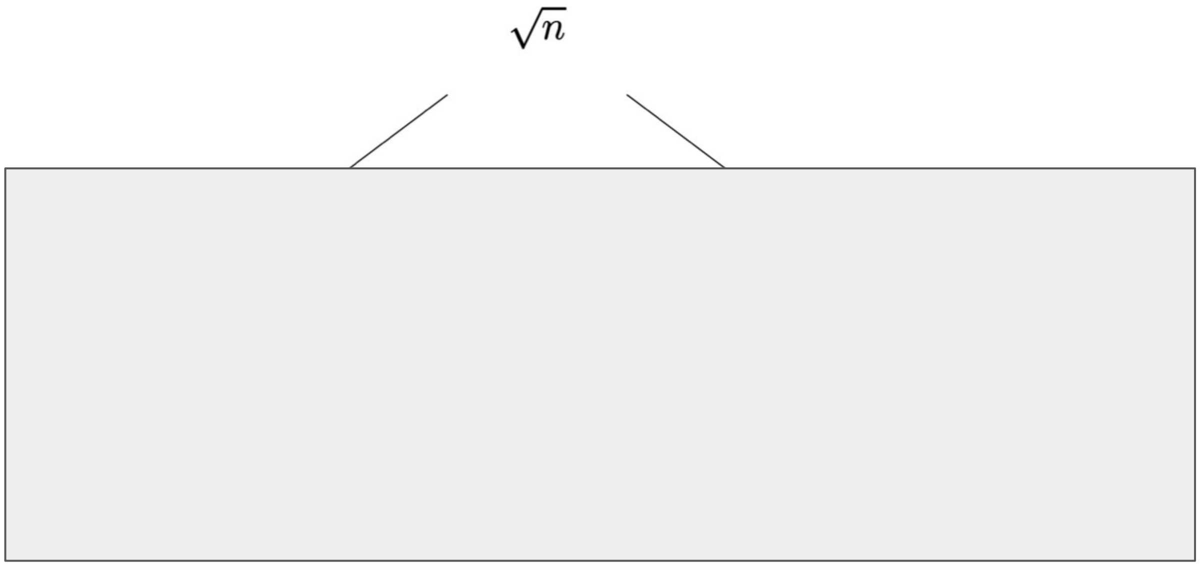

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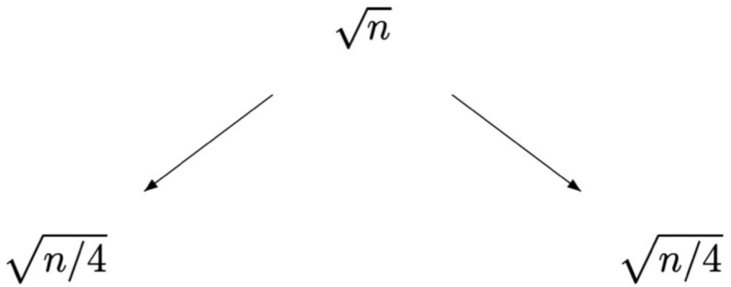

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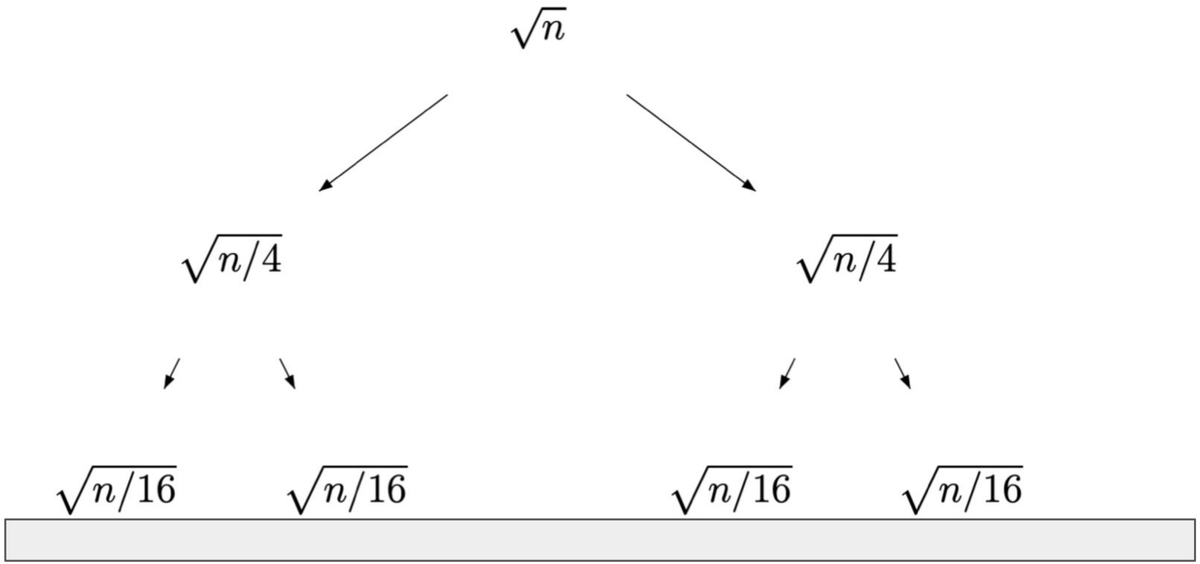

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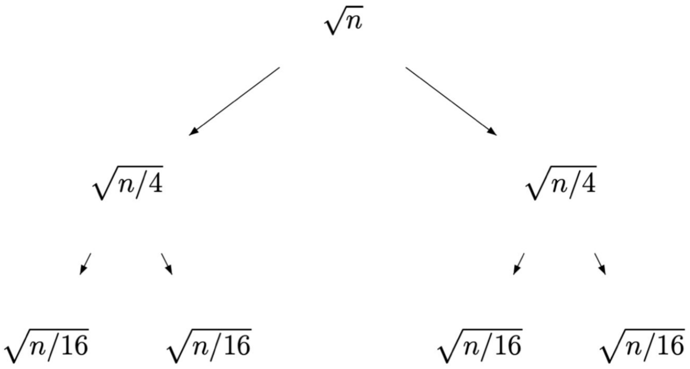

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Cost at first level:
Cost at second level:
Cost at ith level:

\# levels:

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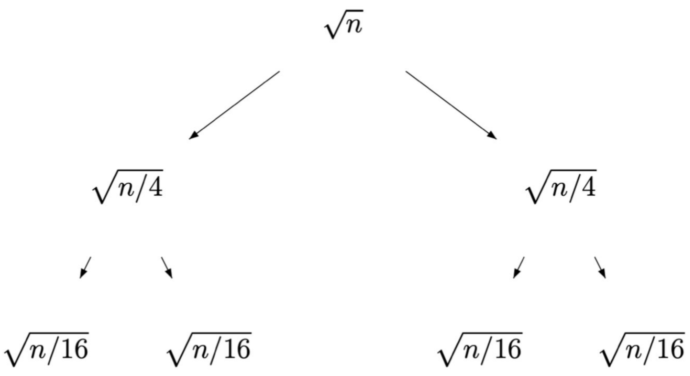

Cost at ith level: $\sqrt{n}$

Number of levels: $\log_4 n$

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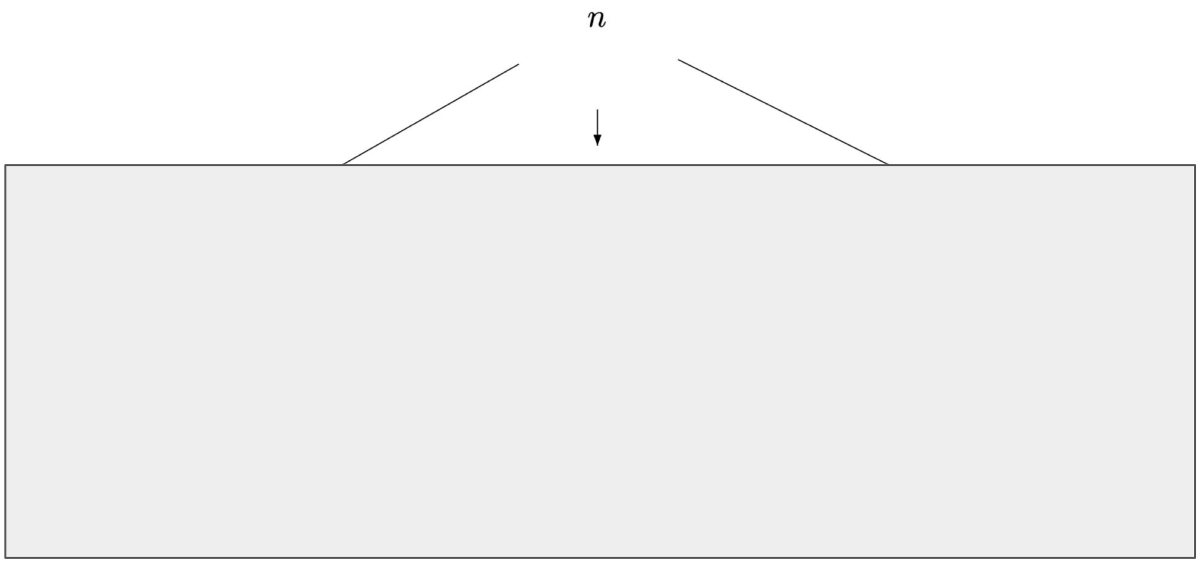

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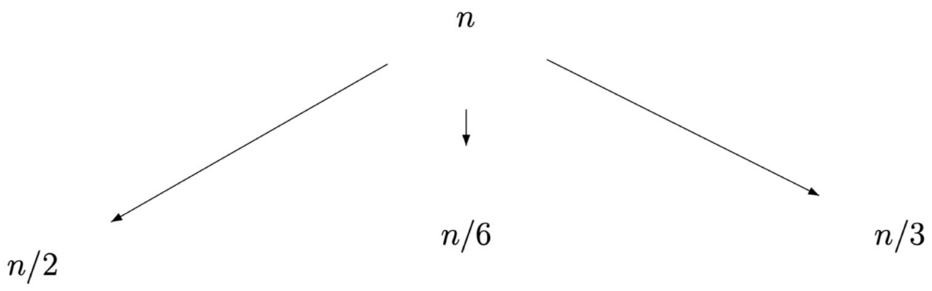

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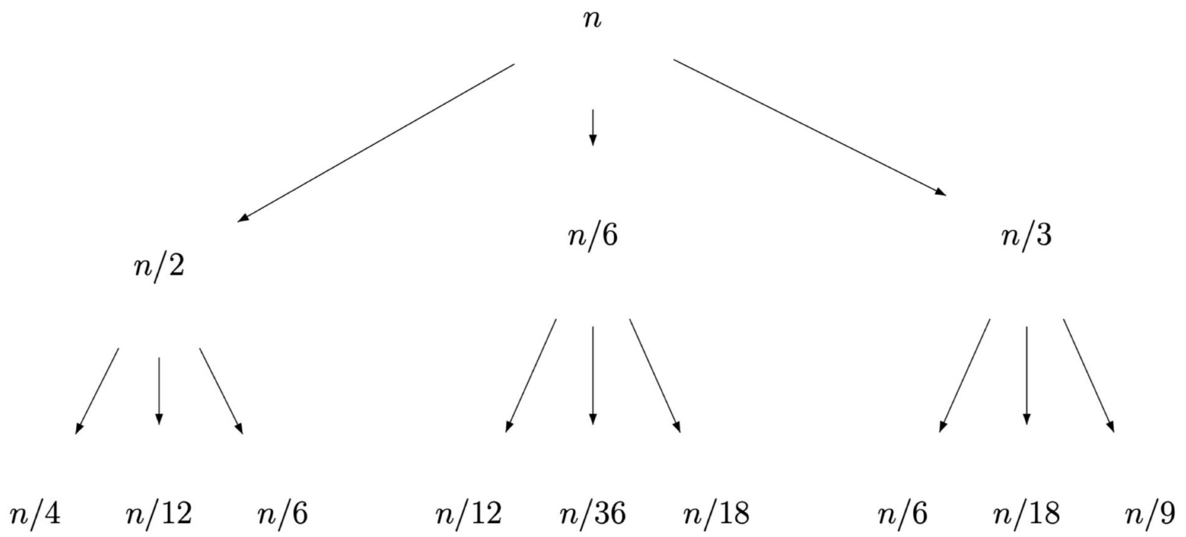

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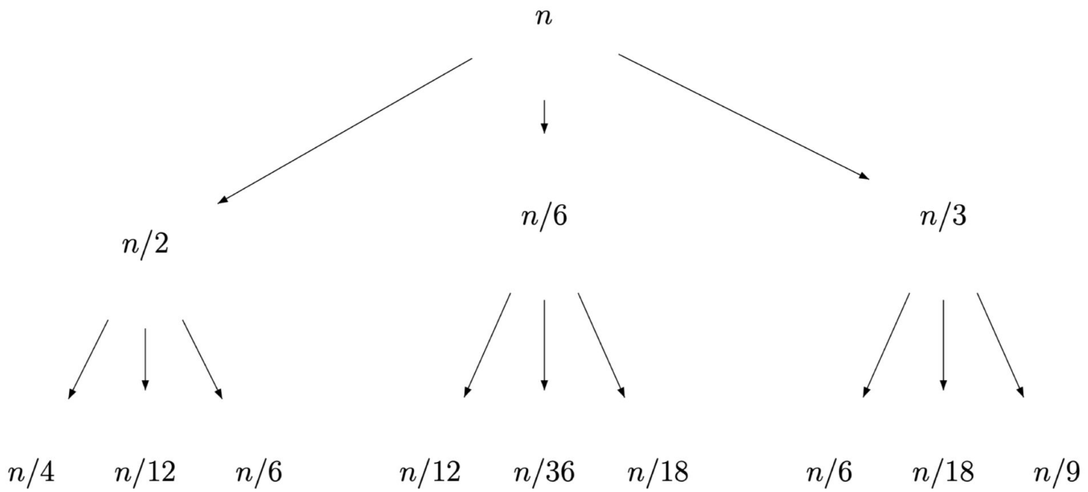

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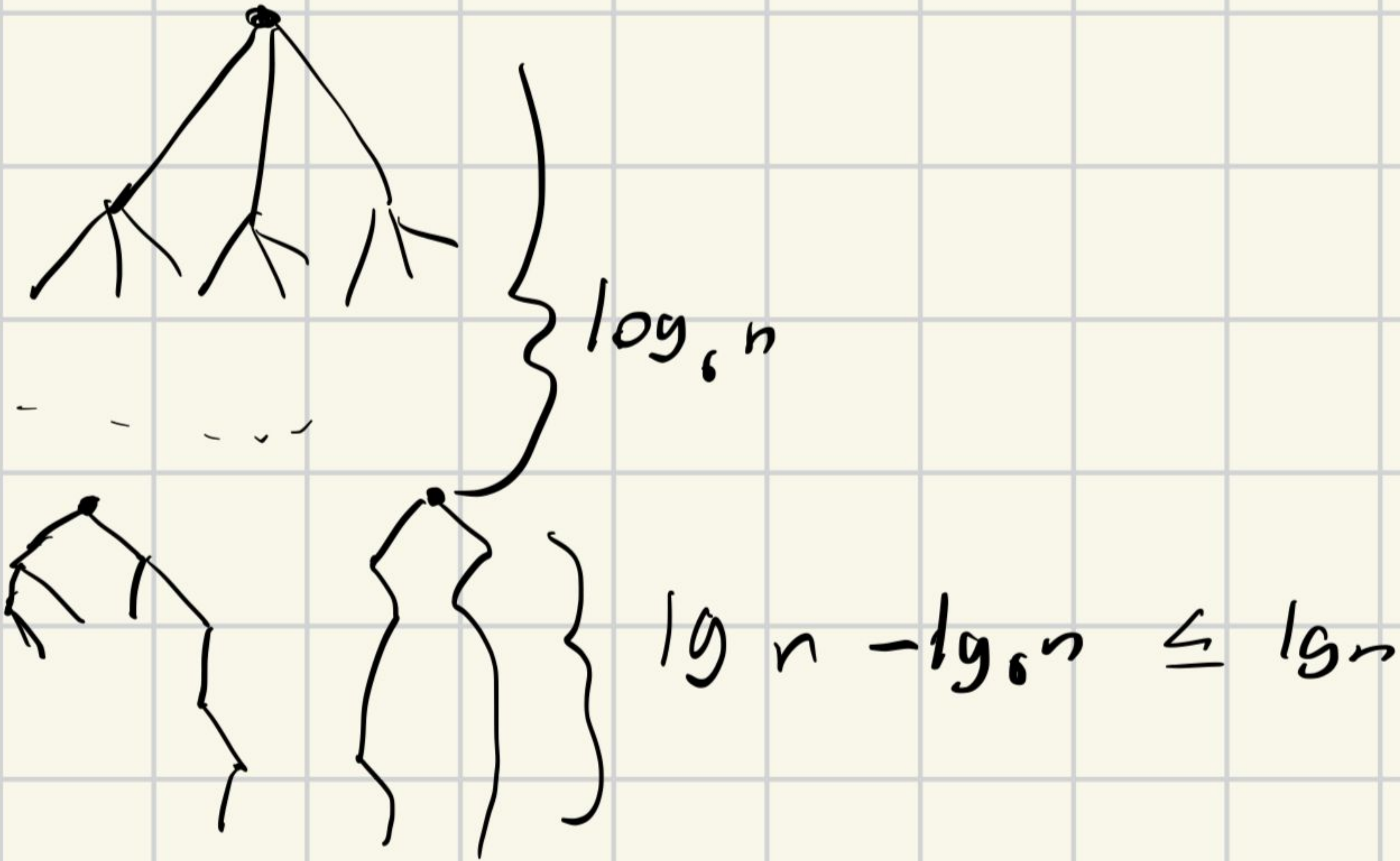

log₆ n
lg n − lg₆ n ≤ lg n

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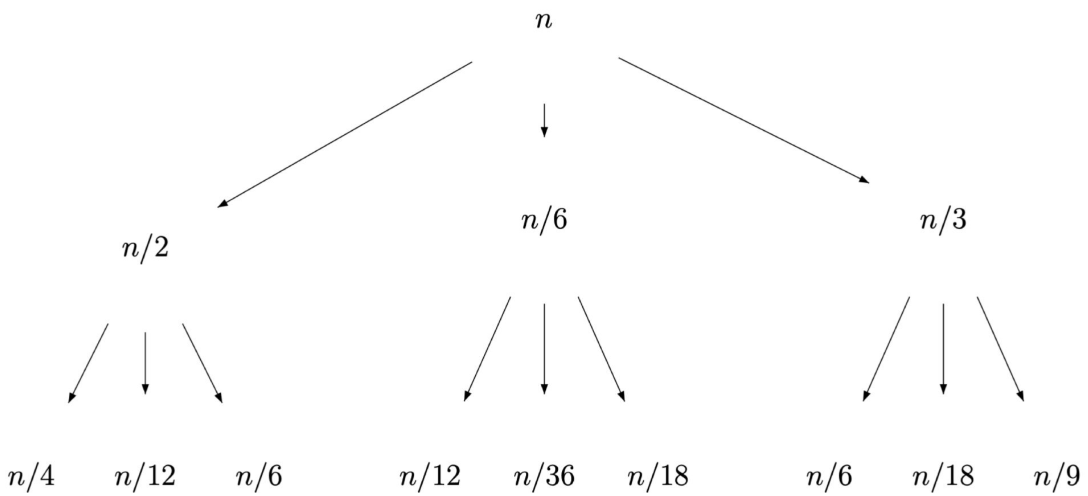

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3. **Derive the sum and closed form**

Cost at ith level: n

Number of levels: $O(\lg n)$

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What is the problem with a tree?

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We usually like recurrences of this form

$$S(n) = \alpha S(n/\beta) + f(n),$$

E.g question 1 recurrences

Solution: Variable change! But to what value?

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Change variable: m =

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Change equation: S(m) =

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This is just merge sort! O(mlogm) = O(log n * (log log n))

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(Algorithm Design) Describe a $\Theta(n \log n)$ algorithm that, given a set S of n integers and another integer x, determines whether or not there exist two elements in S whose sum is exactly x.

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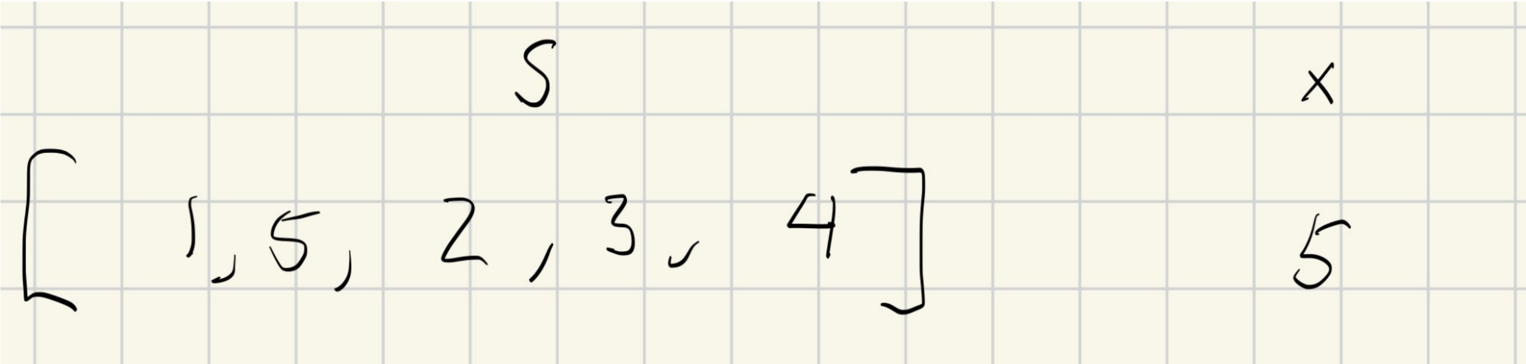
Naive n^2 strategy?



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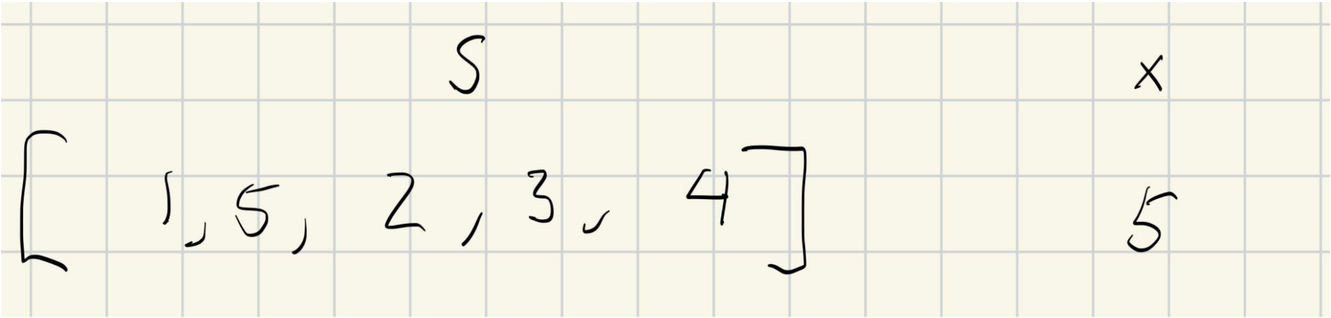

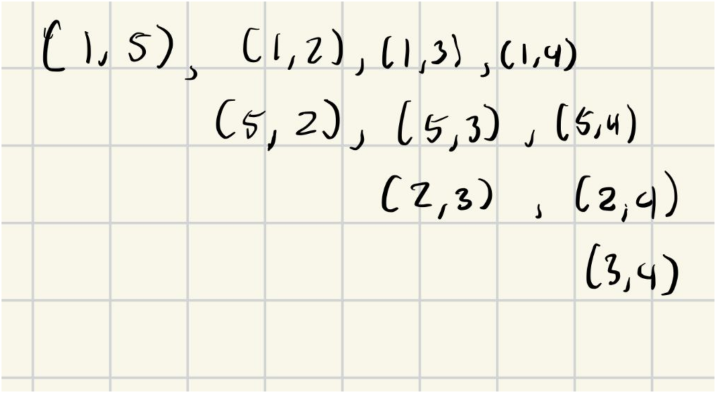

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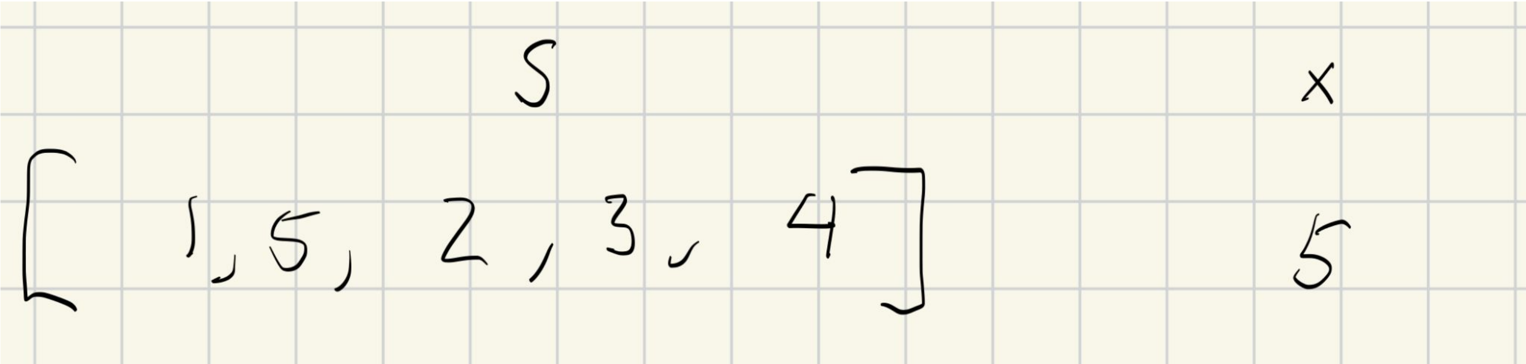
What can we do in nlogn time?



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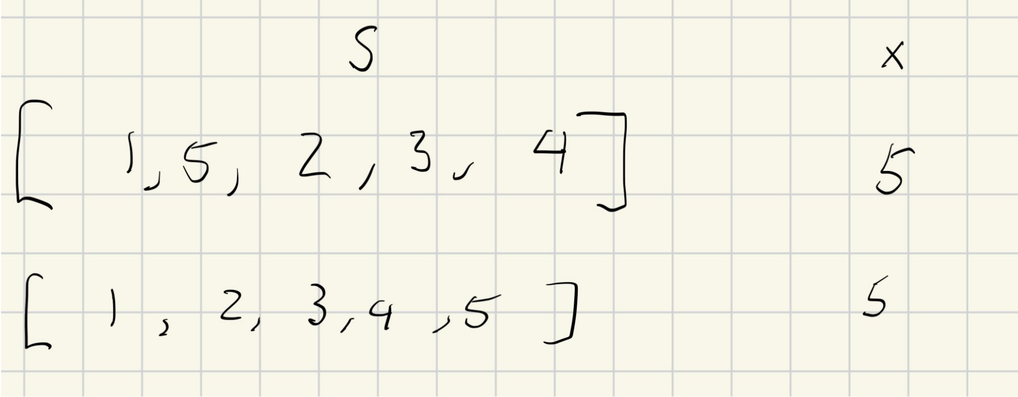

What can we do in nlogn time? Sort

Ok now that elements are sorted,

What do we do?

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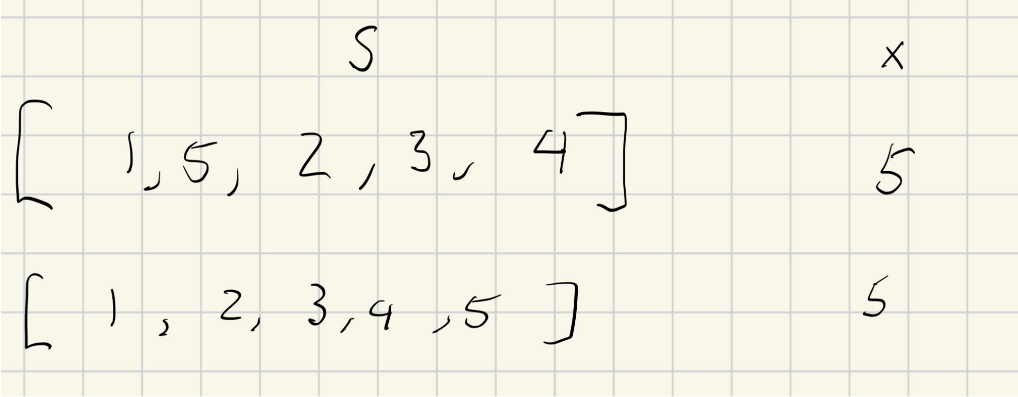

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<u>Idea</u>: find pairs smarter

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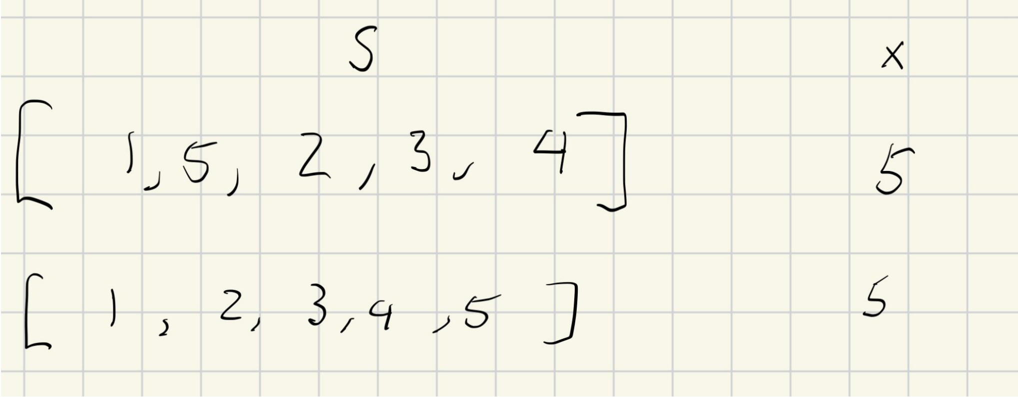

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Idea: find pairs smarter

Hold a left,right pointer, calculate sum

If sum > x: move right pointer

If sum < x: move left pointer

Question 4

(Linked List) Consider a sorted circular doubly linked list of N numbers where the head element points to the smallest element in the list. Provide the asymptotic complexity in big-Θ with a brief explanation (including assumptions and analysis for each case, if there is more than one) for the following operations.

1. Inserting an element in its sorted position.
2. Finding the smallest element in the list.
3. Finding the 3^{rd} - largest element in the list.
4. Finding the median in the list.