Problem Reduction Terminology. For two problems X and Y we will often ask,

> "Show that a polynomial time algorithm solving X implies a polynomial time algorithm solving Y."

If we can do this, then we write "$Y \leq X$."

1. Write down in your own words what "$Y \leq X$" means here.
2. Write down the contrapositive of what you wrote in (1).
3. For some problem X, suppose we can show that X has a poly-time algorithm.
   - (True/False) $\mathsf{3SAT} \leq X$
   - (True/False) $X \leq \mathsf{3SAT}$
   - (True/False) $\mathsf{2SAT} \leq X$
   - (True/False) $X \leq \mathsf{2SAT}$
4. Write down the CNF formula for "$x = y$"

Problem $\mathsf{4SAT} \leq \mathsf{3SAT}$**.** Show that a polynomial time algorithm for 3SAT implies a polynomial time algorithm for 4SAT.

How would you format your answer on an exam?

Problem . Let the HALFSAT problem be defined as follows.

HALFSAT: Given a formula in CNF form with $2n$ variables and no negations, determine whether the formula can be satisfied by setting at most n variables to TRUE.

Here is an example of a HALFSAT instance:

The formula

$$X = (x_1 \vee x_5) \wedge (x_2 \vee x_3 \vee x_6) \wedge (x_1 \vee x_4) \wedge (x_4) \wedge (x_2 \vee x_5)$$

can be satisfied by setting x_2, x_4, x_5 to TRUE (and x_1, x_3, x_6 to FALSE).
Thus, X is an instance of HALFSAT.

Prove that a poly time algorithm for HALFSAT would imply a poly time algorithm for 3SAT (That is, 3SAT $\leq$ HALFSAT).