

Problem Coupon Collecting. Discount Den is having a promotional sale where every purchase comes with a random coupon, where there are n distinct coupons. Once you collect all the n distinct coupons, the owner gives you the land deed to the store, and you become the new owner.

Formally, there are n different types of coupons. Each item you buy comes with a random type of coupon. What is the expected number of coupons needed until you own Discount Den?¹

¹Hint: Say we have one coupon so far. What is the probability we draw a new coupon?

Ethan's Grading Problem The final results are back, and the 381 GTAs need to grade a whopping n student submissions! They want to spend as little time as possible grading (they have better things to do!). They can choose between two methods for grading any given submission:

- **Real grading:** This always takes a couple of painstaking minutes per submission, but it guarantees an accurate grade. The solution type will also be cached into the GTAs brain.
- **Skimming:** This method takes negligible time. However, the grader will skim so quickly that they only have a 50% chance of giving an accurate grade! That is, of course, unless they've seen the solution type before (in which case it is guaranteed with 100% chance to be accurate).

Unfortunately, the students have many different solution types. A preliminary ChatGPT analysis showed that every solution type had at least k instances among the n submissions.

Give a randomized algorithm to minimize the total number of submissions that were *real graded* such that the probability of grading any submission inaccurately is at most $1/n$ (the GTAs can handle a couple regrade requests, after all). Express the asymptotic bound of real graded submissions in terms of n and k .